Rooting spectroscopy in QCD

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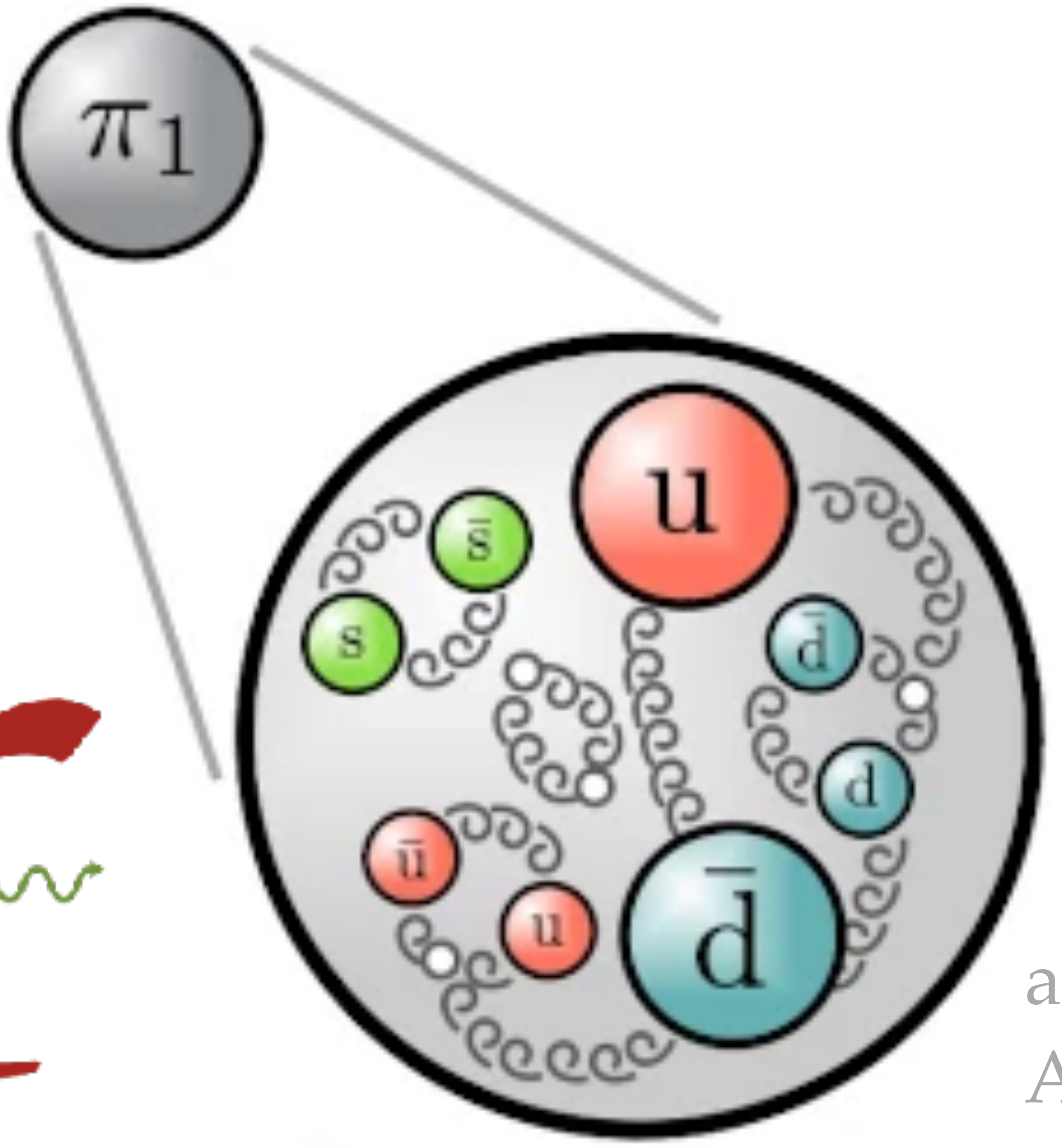
EXOHAD

EXOTIC HADRONS TOPICAL COLLABORATION

Rooting spectroscopy in QCD

A Lattice QCD program running in parallel with experiments is critical.

- ☑ **Guide experimental searches** [e.g. π_1],
- ☑ **Confirm existence** [e.g. tetraquarks, pentaquarks],
- ☑ **Understand their nature** [*observations are not enough*!].



artistic rendition by A. Jackura

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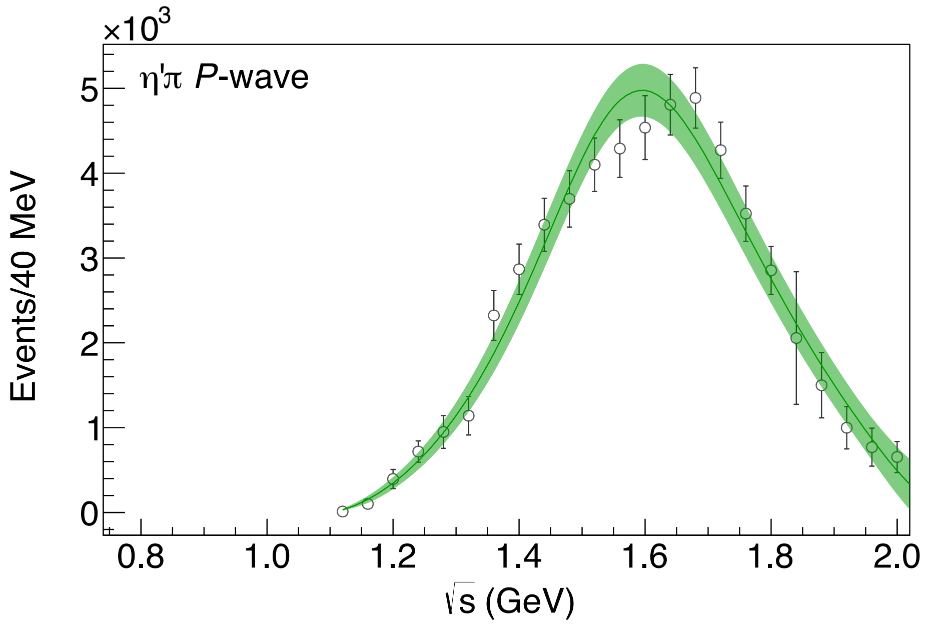
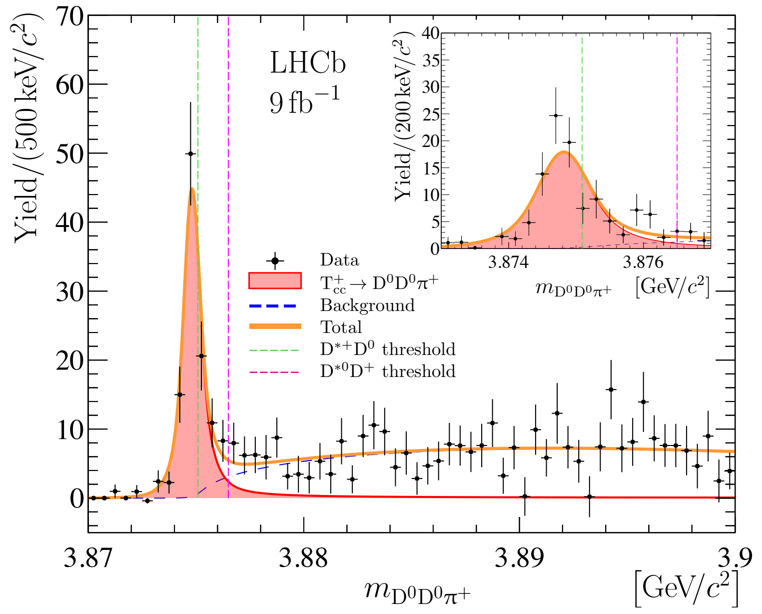
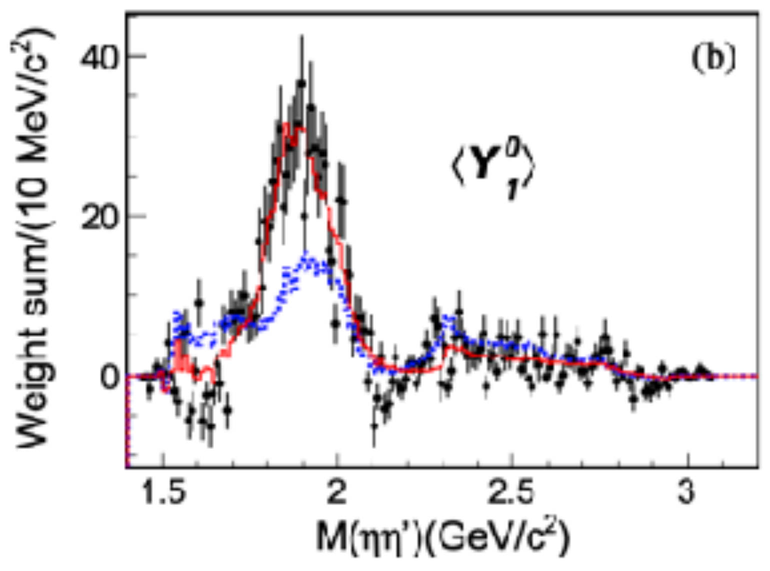
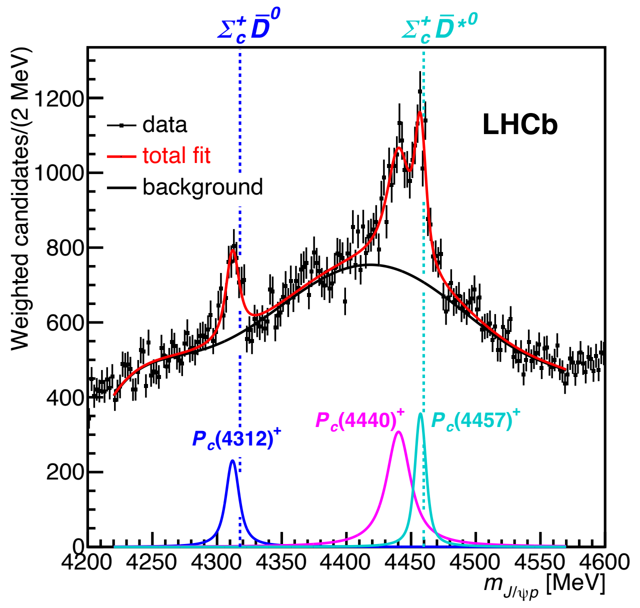
- **resonances** or multi-body **bound states**,
- **dynamical enhancement** in scattering amplitudes.

Numerous experimental searches…









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Challenges to overcome:

- ☐ scattering amplitudes are direct inaccesible via lattice QCD,
- ☐ **formalism needed to access amplitudes,**
- ☐ **increasingly complicated analysis,**
- ☐ **warm bodies,**
- ☐ …

ExoHad
Exotic Hadrons Topical Collaboration
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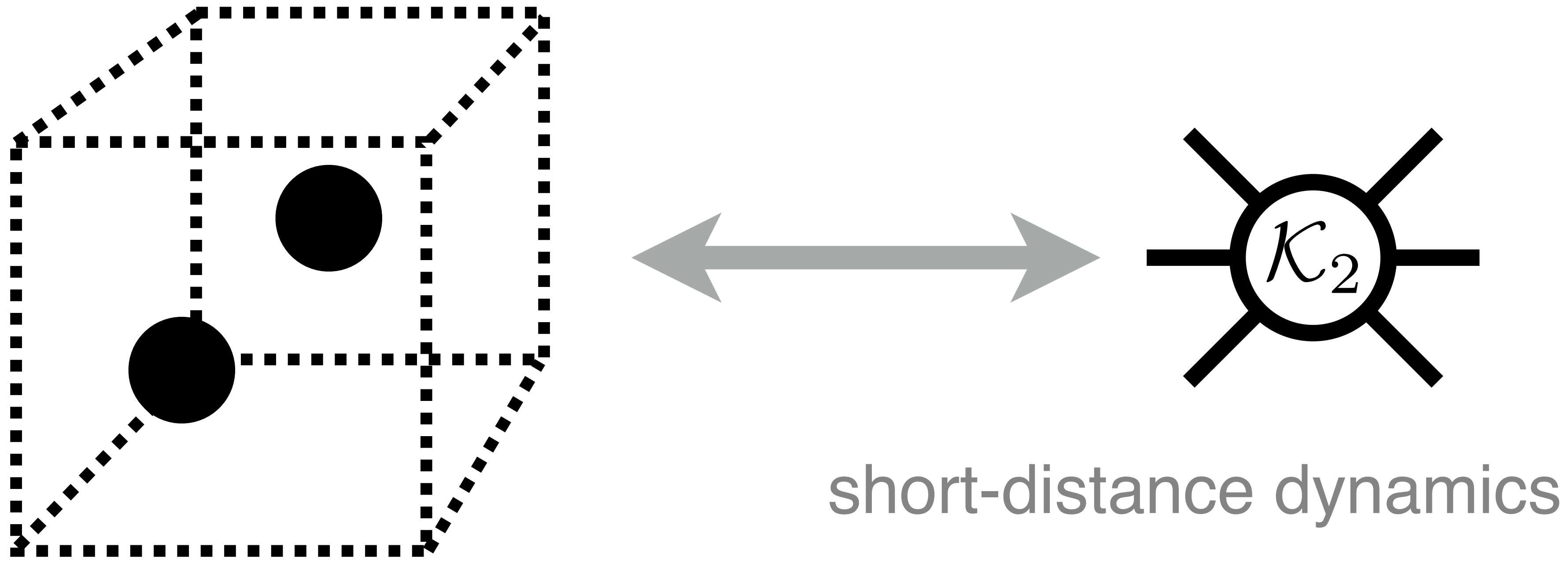
Lattice QCD milestones

- ☑ First dispersive extractions of the $\sigma/f_0(500)$ from lattice QCD at various light-quark masses
- ☑ Extension of finite-volume three-body formalism to scattering of particles with spin
- ☐ Lattice QCD calculation of the 3π scattering amplitude when 2π can form the ρ resonance
  - ☑ Analytic continuation of three-body integral equations, ← [year 3 milestones]
  - ☑ Partial wave projection of OPE,
  - ☐ Integral equations for coupled-channel systems,
  - ☐ Code development for finite-volume analysis.

Key requirements

- [x] Generalized eigenvalue problem (GEVP),
  - [ ] large basis: $\mathcal{O}_b \sim \bar{q}\,\Gamma_b\,q, \pi\pi, K\overline{K}, \ldots, 4\pi, \ldots$
  - [ ] contractions: $C_{ab}^{2pt.}(t,\mathrm{P}) \equiv \langle 0 | \mathcal{O}_b(t,\mathrm{P})\mathcal{O}_a^\dagger(0,\mathrm{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t}$
  - [ ] "diagonalization".

- [x] Formalism:



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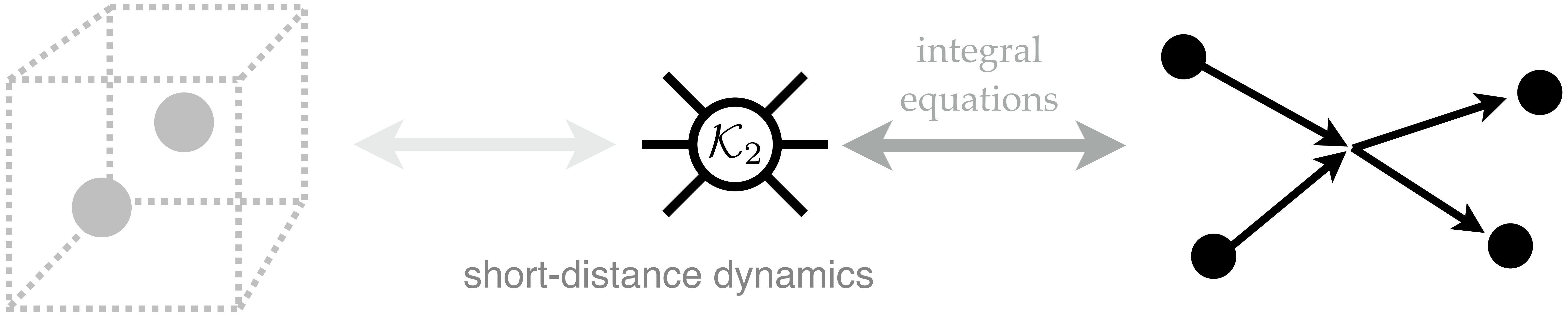

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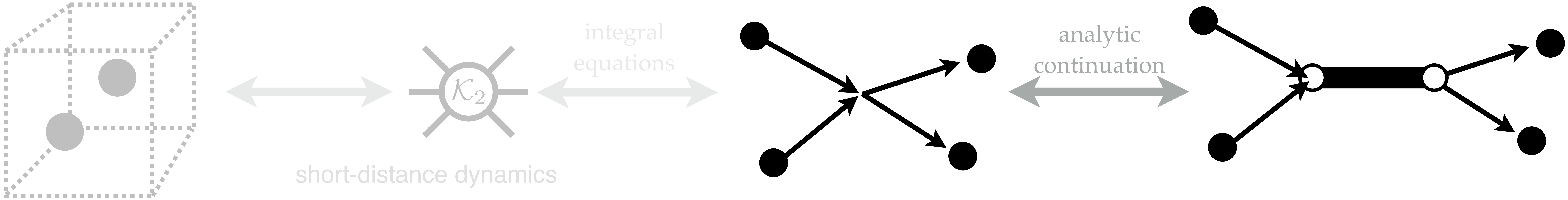

$$\det\left[F^{-1}(P,L)+\mathcal{K}_2\right]=0$$

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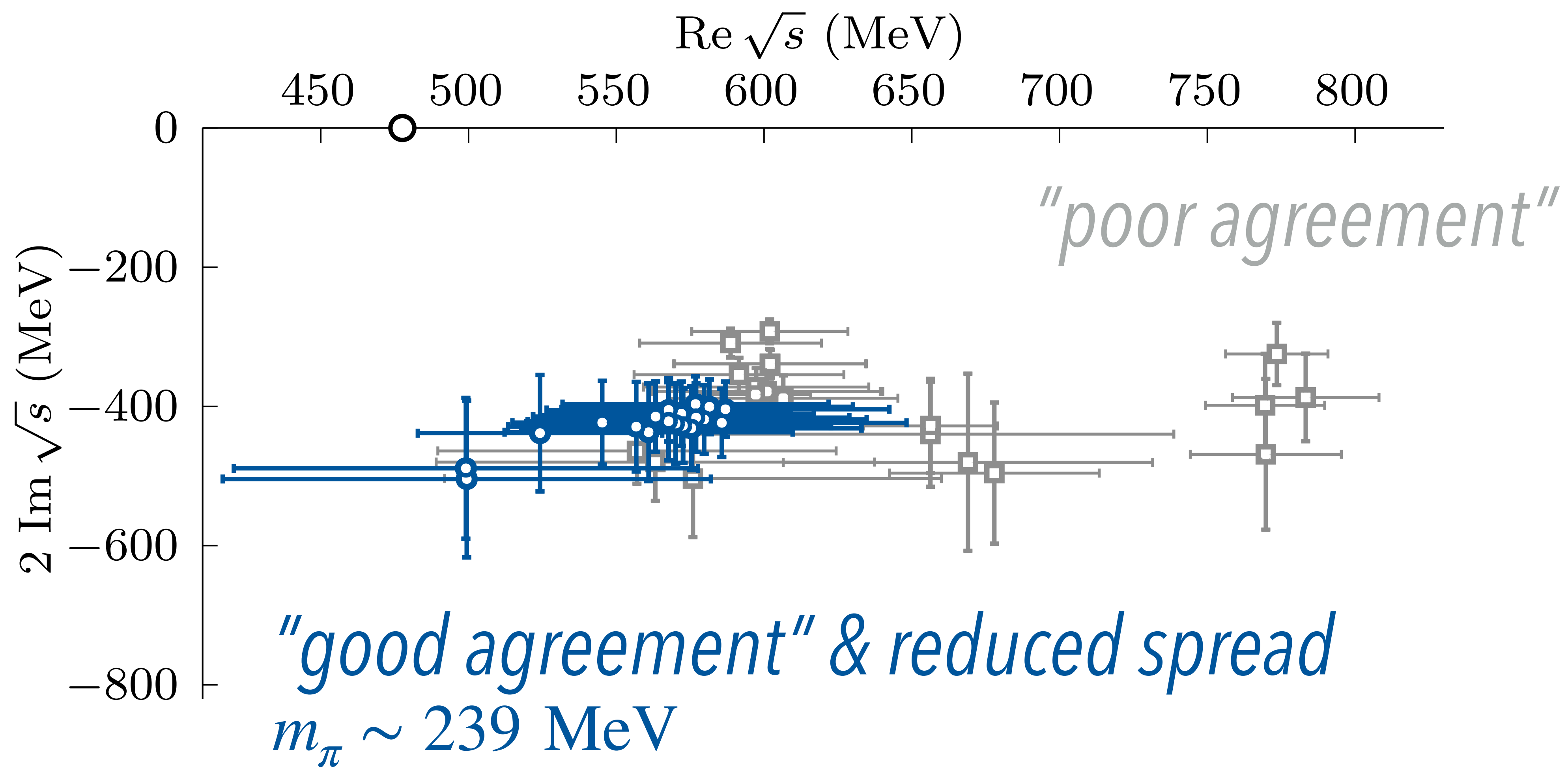
$$\mathcal{M}_2 = \frac{-g^2}{s-\left(m-i\frac{\Gamma}{2}\right)^2}$$

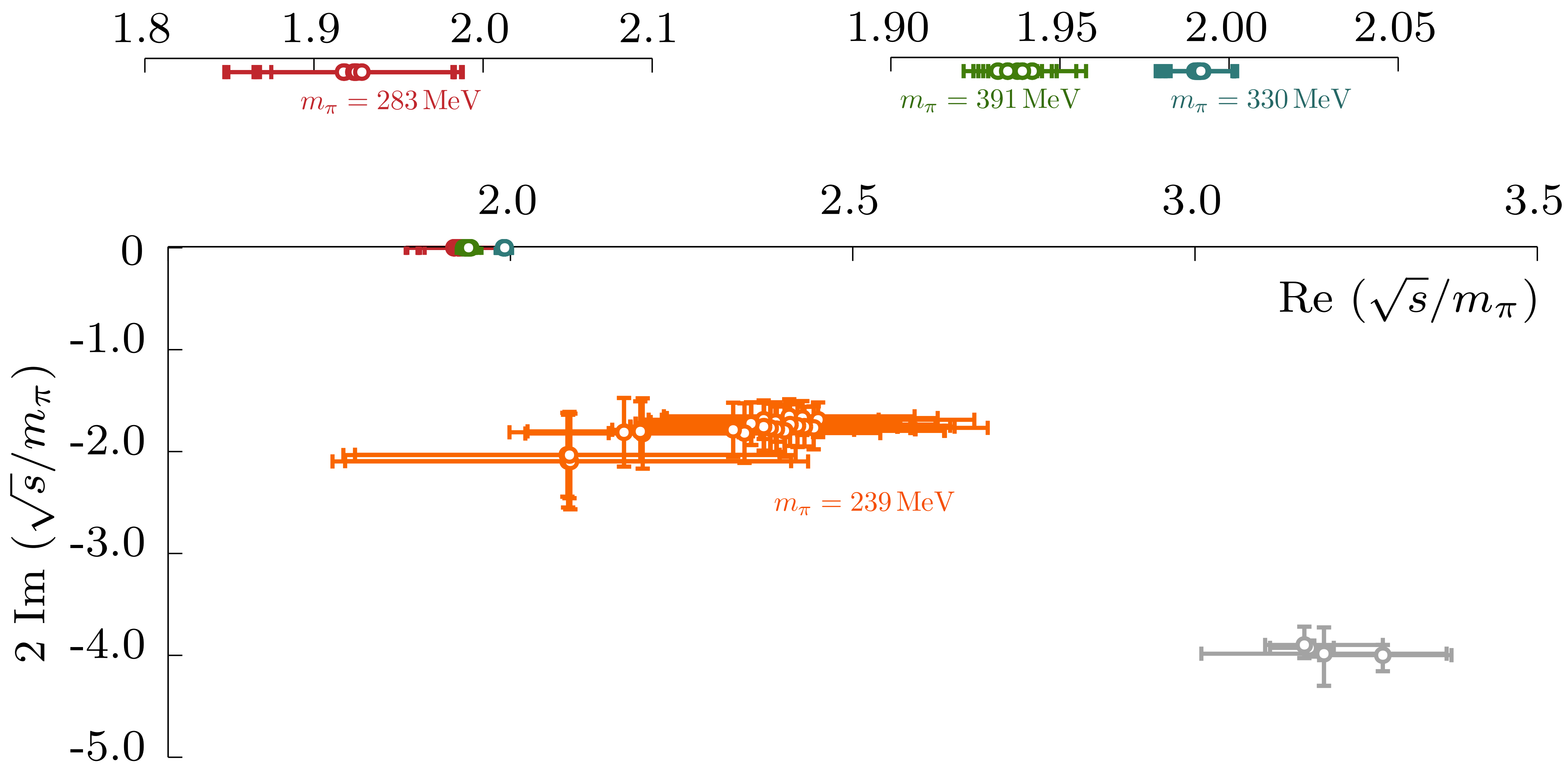
$\pi\pi$ and the σ imposing crossing symmetry

- For $\pi\pi$, Roy Equation can used to test crossing symmetry,

$$\widetilde{\mathcal{M}}^{I}_{\ell}(s) = c^{I}_{\ell}(s) + \int_{4m_\pi^2}^{\infty} ds'\, B^{II'}_{\ell\ell'}(s',s)\, \mathrm{Im}\, \mathcal{M}^{I'}_{\ell'}(s')$$

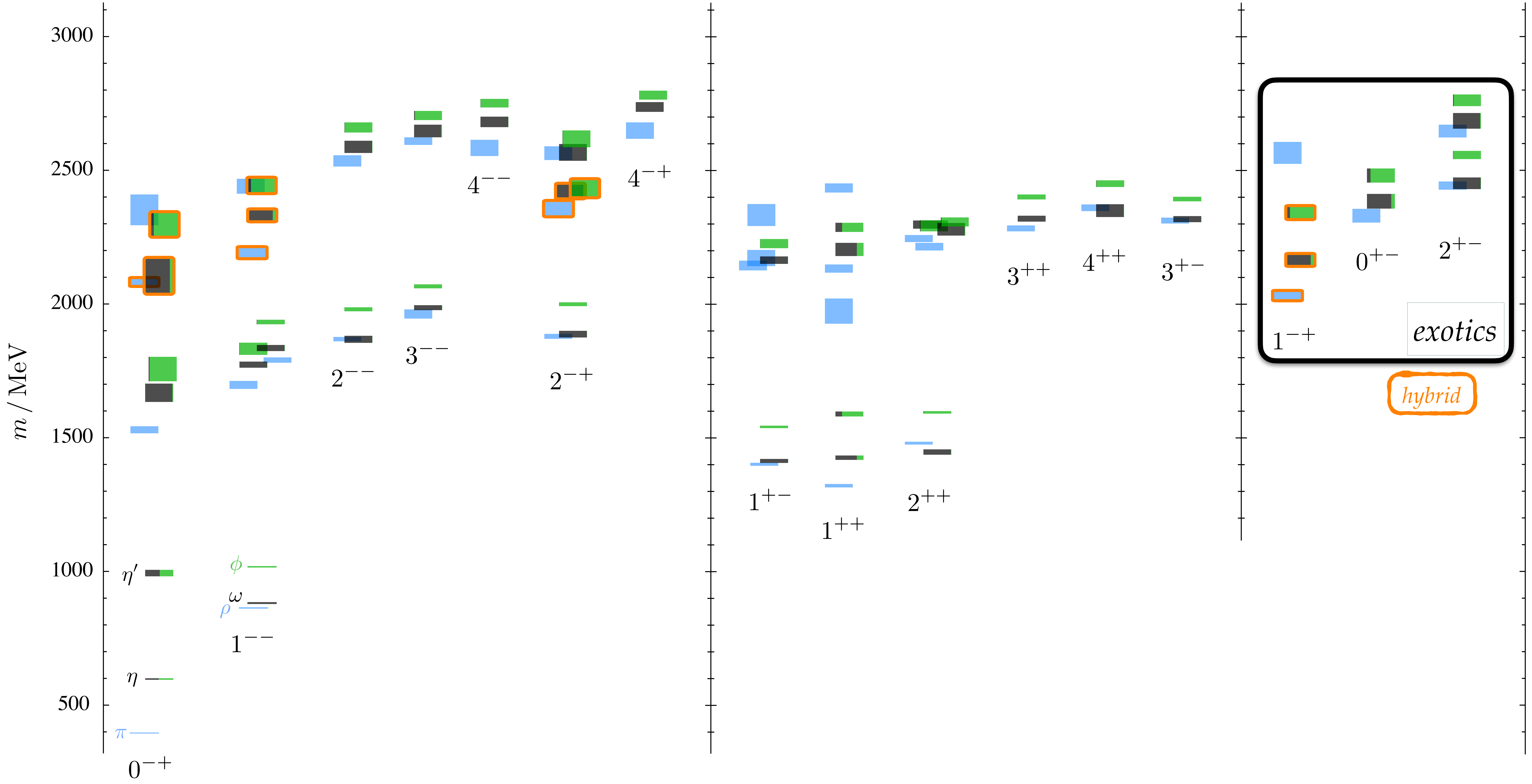
- Amplitudes are discriminated to have good or poor agreement with Roy Eq.,
- this discriminant can be applied for to the σ pole.





Rodas, Dudek, Edwards (2023)

Exotics and three-body decays



m_π=391 MeV

Dudek, Edwards, Guo, Thomas (2013)

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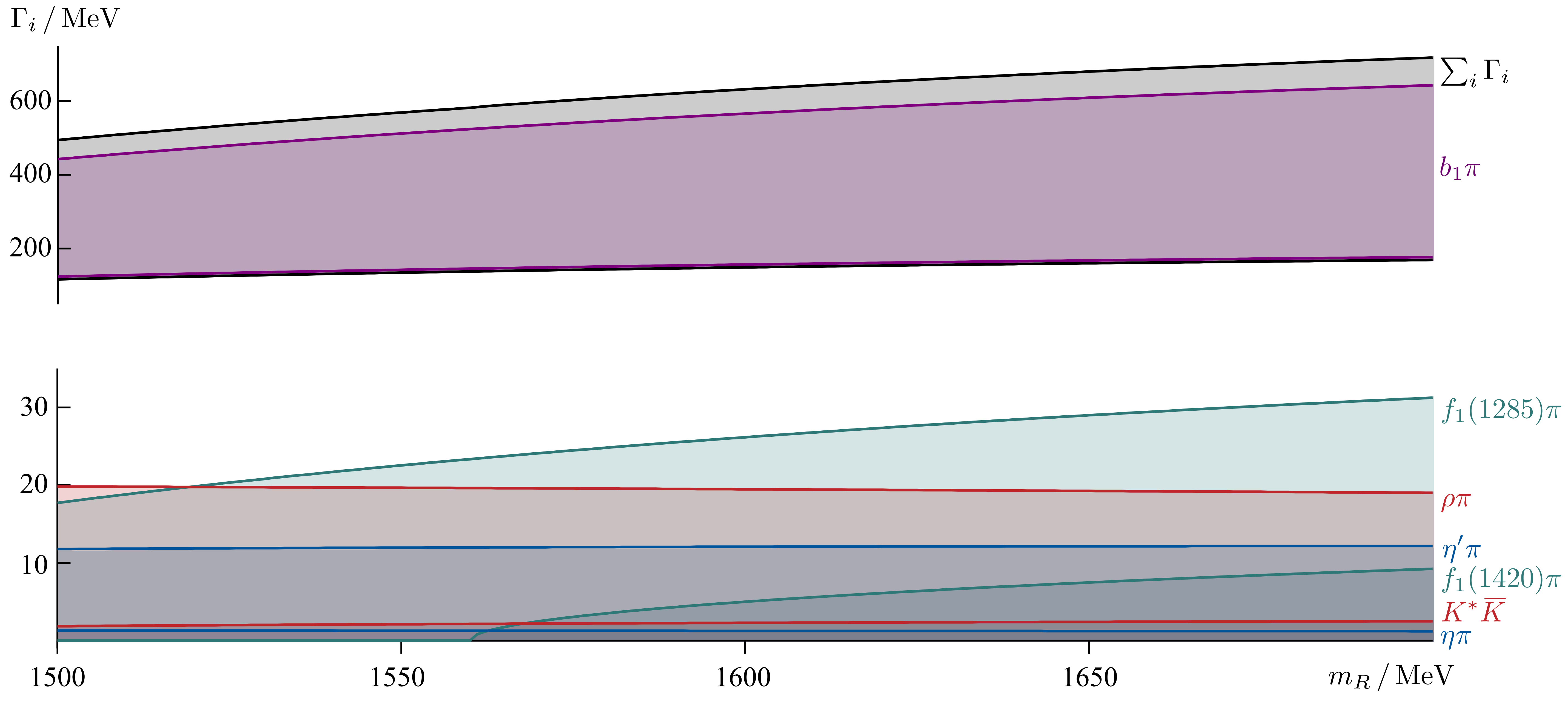
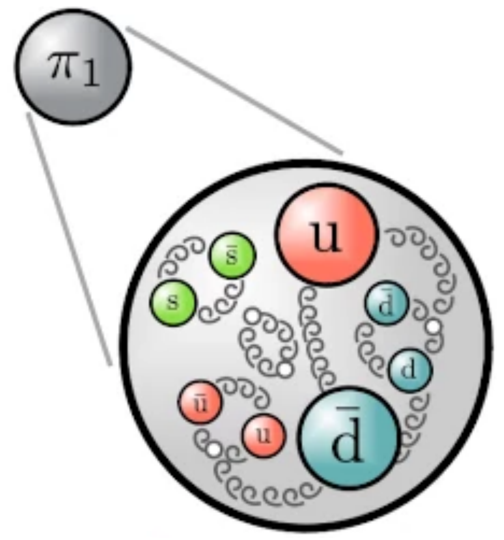
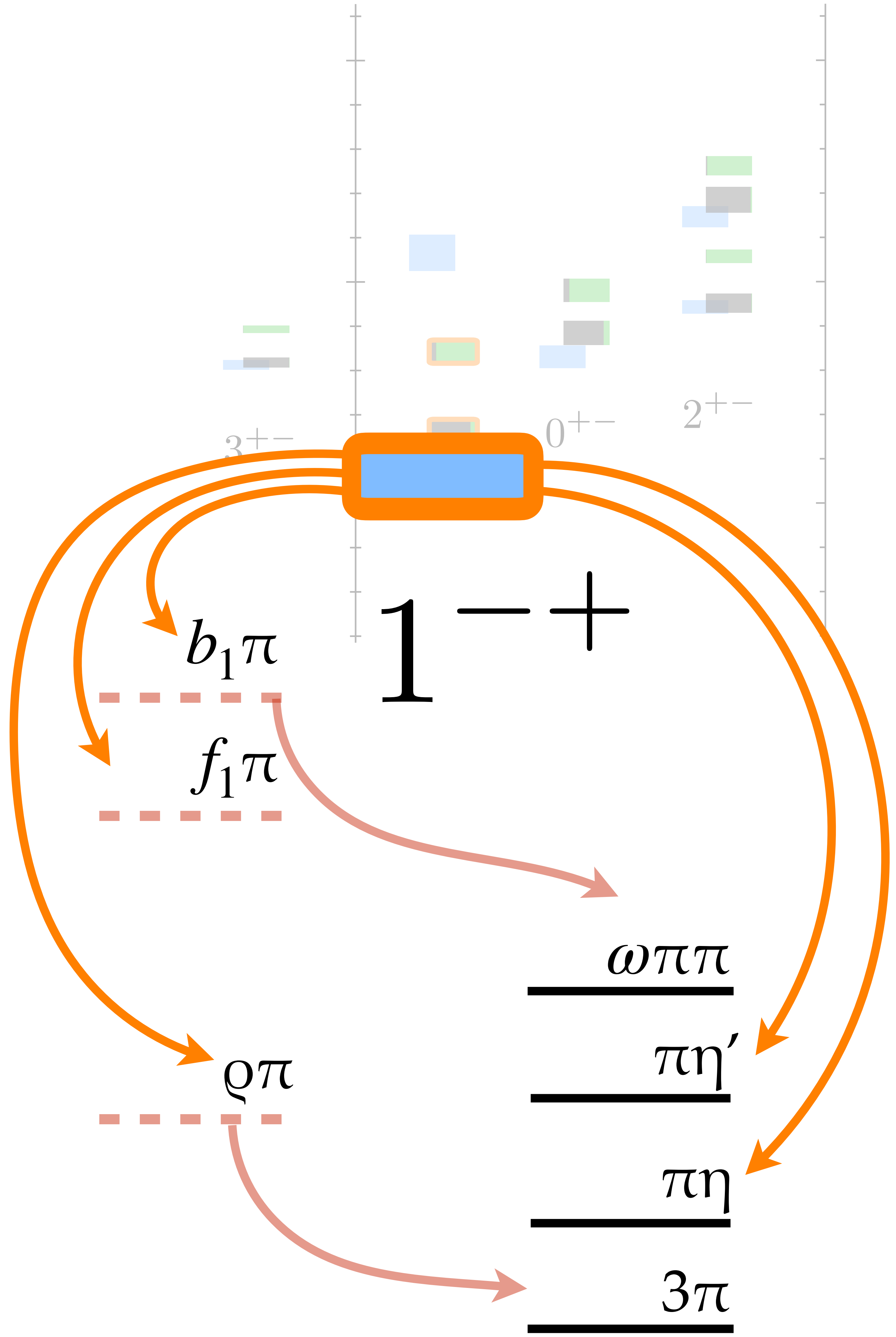
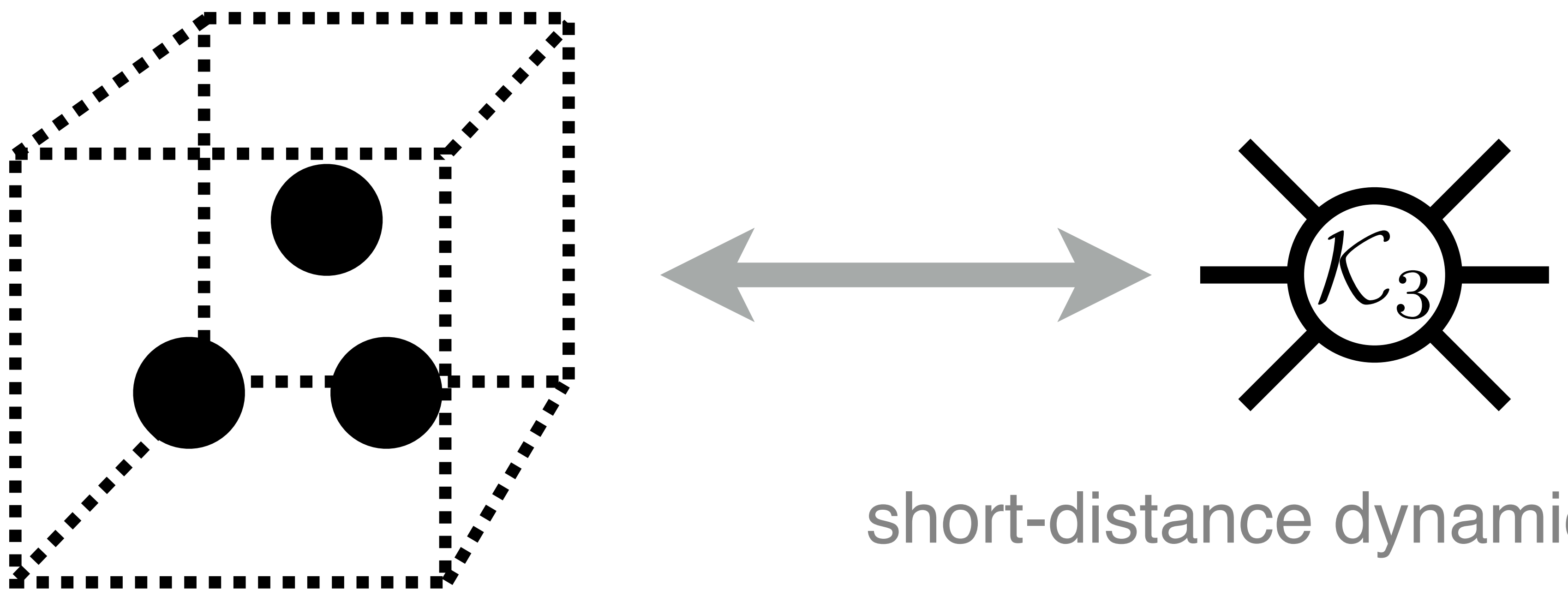






Dudek, Edwards, Guo, Thomas (2013) Woss, Dudek, Edwards, Thomas, Wilson (2020)

Three-body reactions from QCD



short-distance dynamics

$$\det\left[F_3^{-1}(P,L)+\mathcal{K}_3(P^2)\right]=0$$

$3n$ - Draper, Hansen, Romero-López, Sharpe

$DD\pi$ - Hansen, Romero-López, Sharpe

$\eta\pi\pi - \pi K\overline{K}$ - Draper, Sharpe

← [year 1 milestones]

Three-body reactions from QCD

"biggest limitation to date"

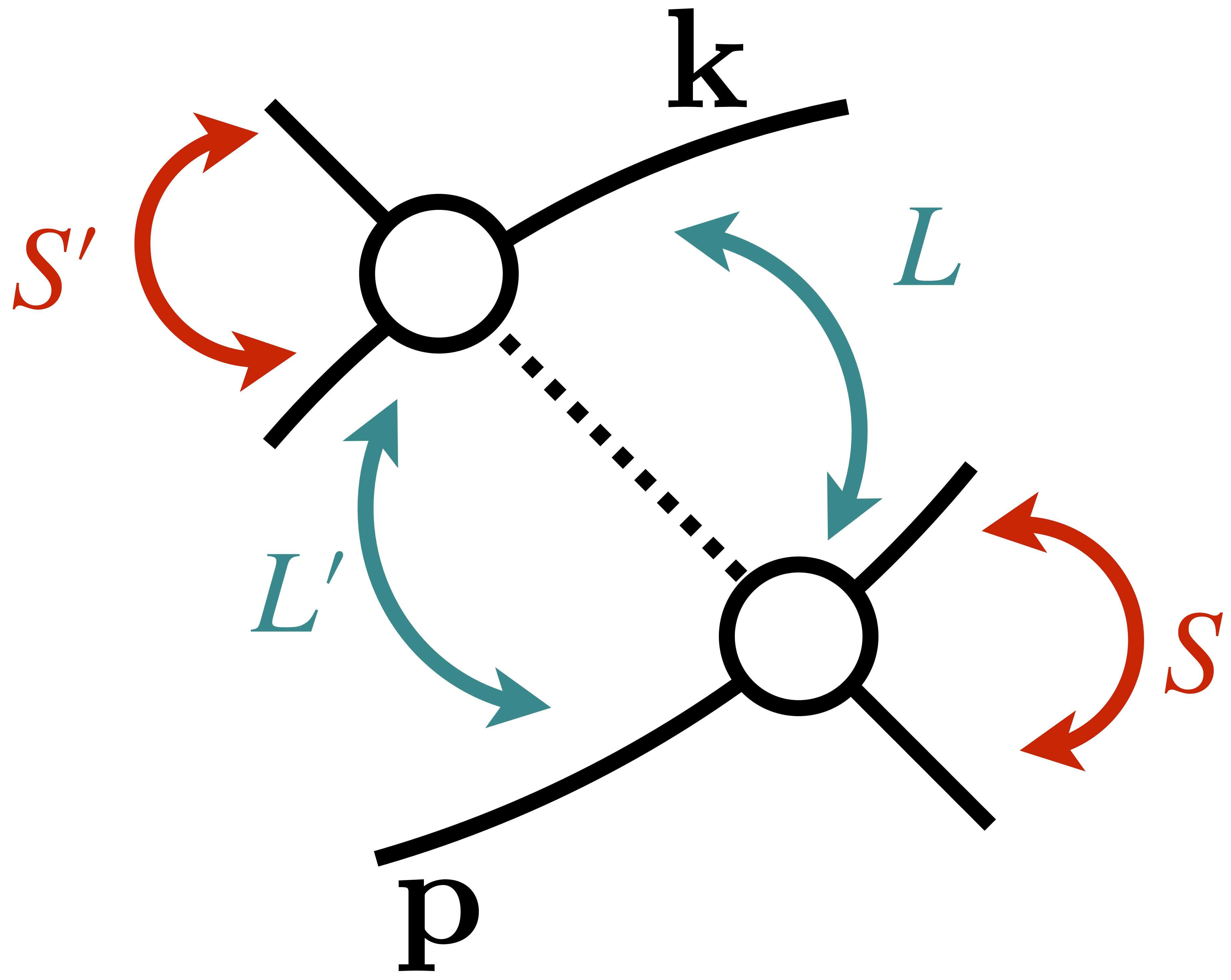
$$i\mathcal{M}_3 = i\mathcal{D} + i\mathcal{M}_{3,\mathrm{df}}\left[\mathcal{D}, \mathcal{K}_3\right]$$

Satisfies integral equations: $i\mathcal{D} = i\mathcal{M}_2 iG i\mathcal{M}_2 + \int i\mathcal{M}_2 iG\, i\mathcal{D}$

Challenges resolved:

- ☑ Partial projection **angular momentum and parity**,
- ☑ numerical solutions
- ☑ analytic continuation. ⟵ [year 3 milestones]

Partial wave projection



$$\left[\mathcal{G}^{J^P}\right]_{L'S',LS} = \left[\mathcal{K}_{\mathcal{G}}^{J^P}\right]_{L'S',LS} + \left[\mathcal{T}^{J^P}\right]_{L'S',LS} Q_0(\zeta_{pk})$$

known kinematic functions

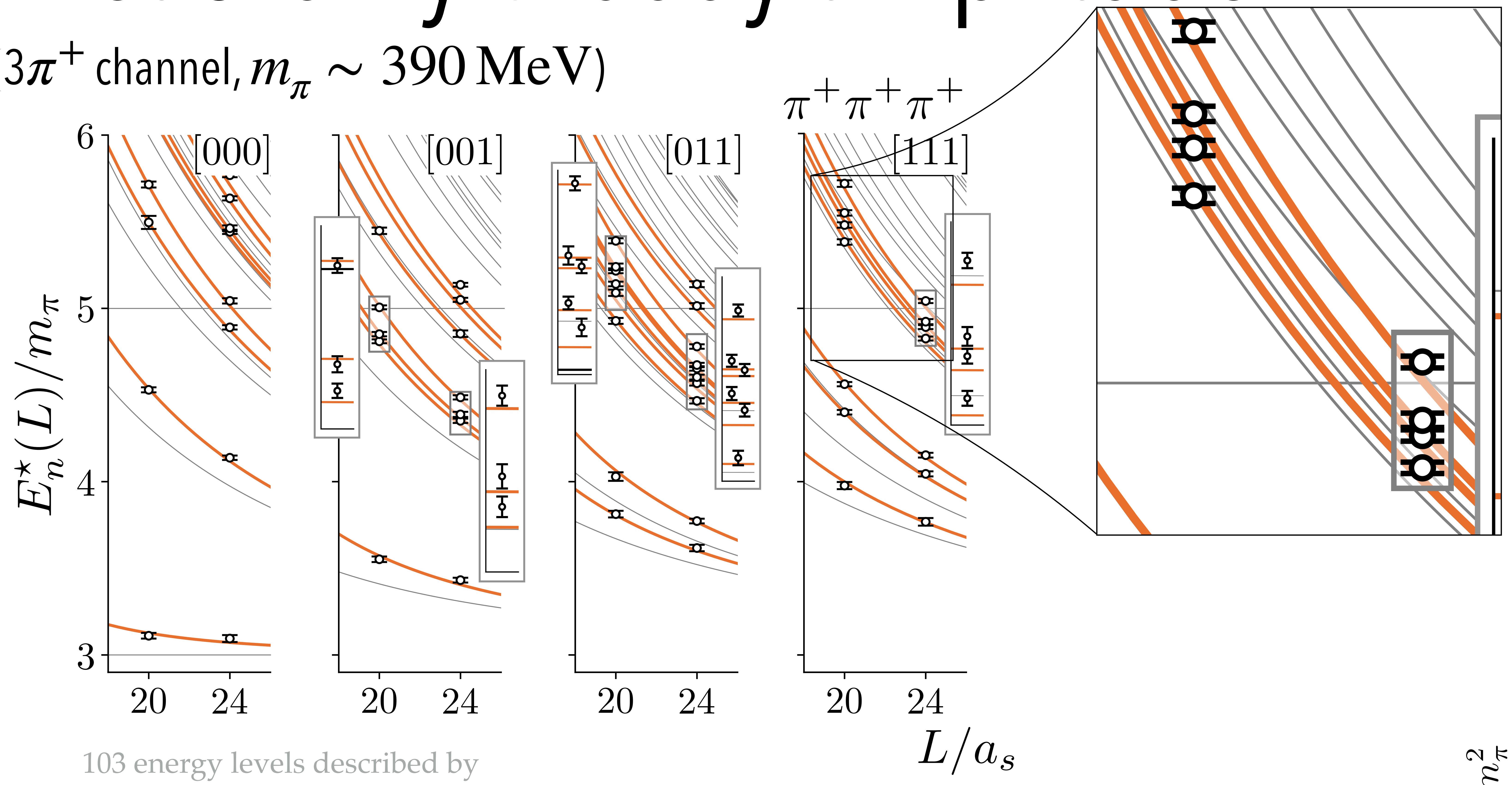
Legendre functions

Jackura, RB (2023)

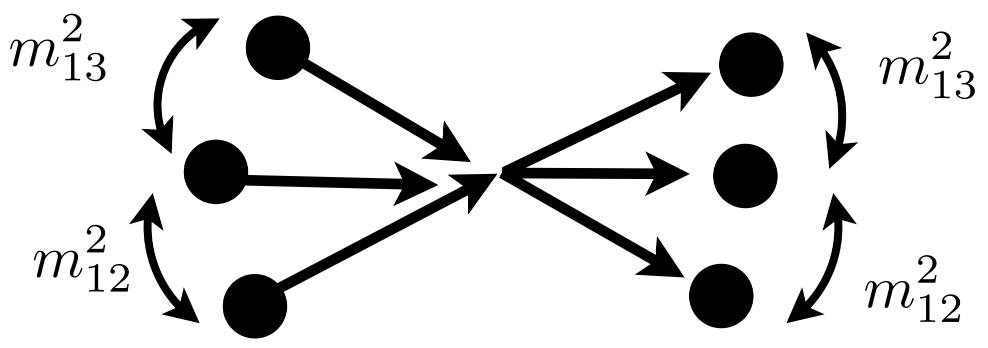
$$Q_0(\zeta) = \frac{1}{2}\log\left(\frac{\zeta+1}{\zeta-1}\right)$$

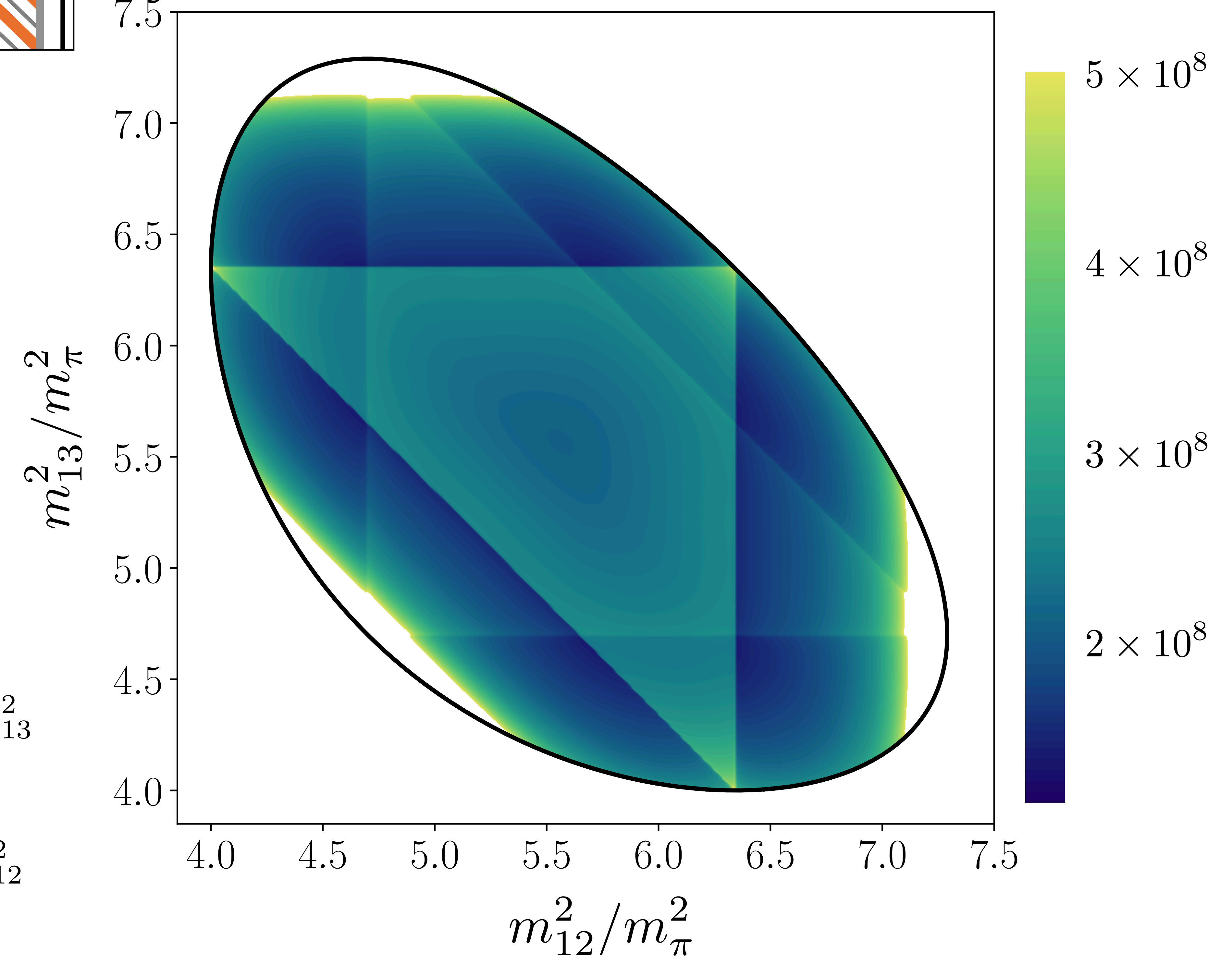
First & only 3-body amplitude

($3\pi^+$ channel, $m_\pi \sim 390\,\mathrm{MeV}$)



103 energy levels described by three numbers: m_π, $a_{\pi\pi}$, $\mathscr{K}_{3,\text{iso}}$

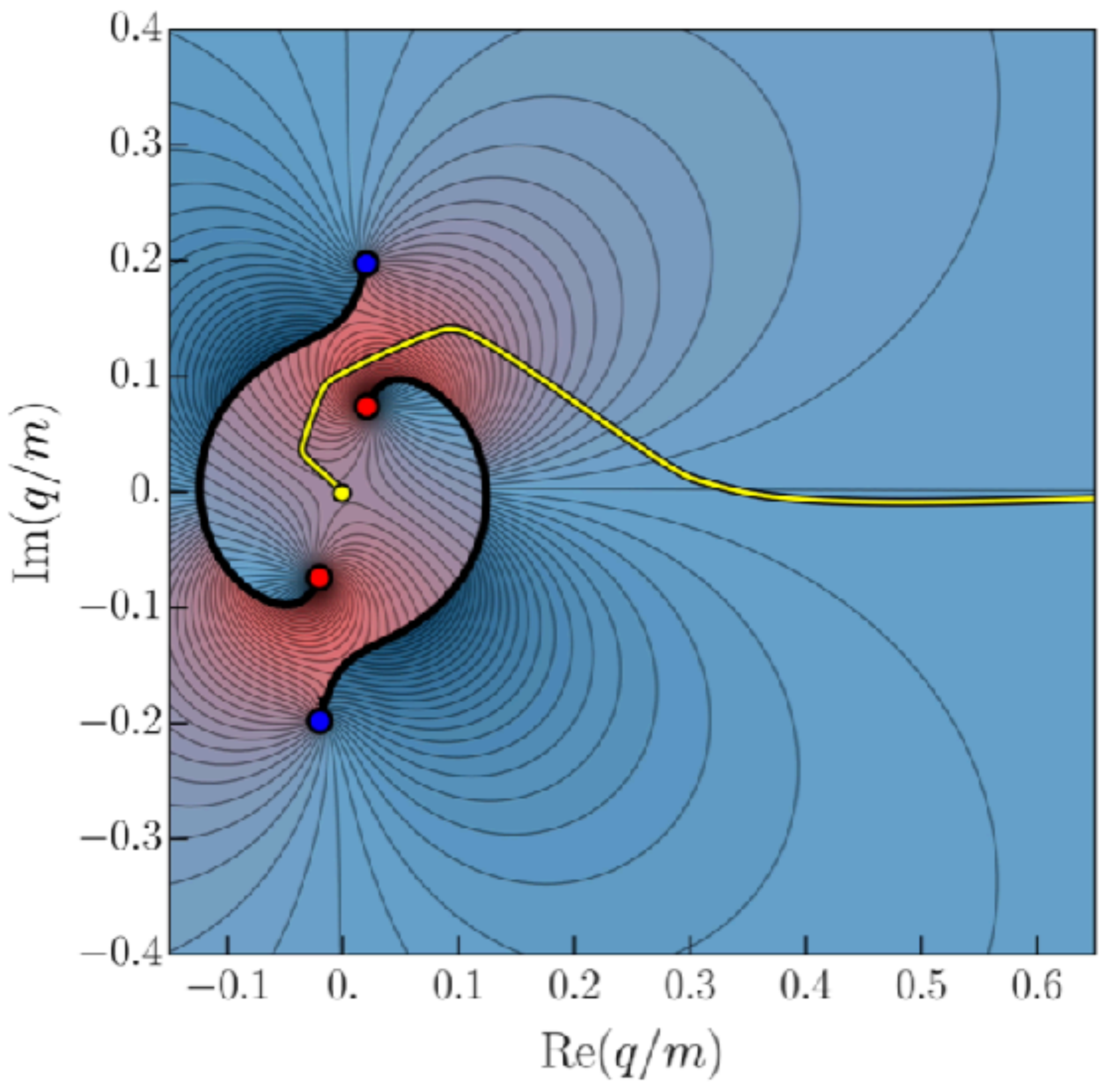




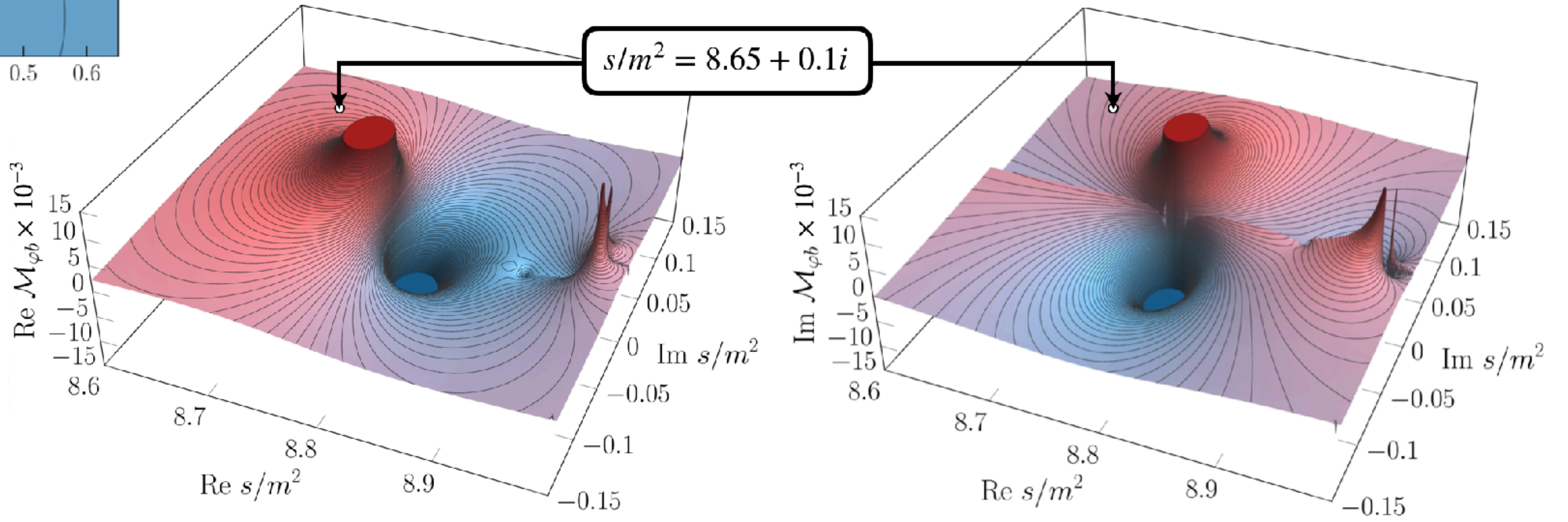
m_π=391 MeV

Hansen, RB, Edwards, Thomas, & Wilson (2020)

Analytically continuing amplitudes

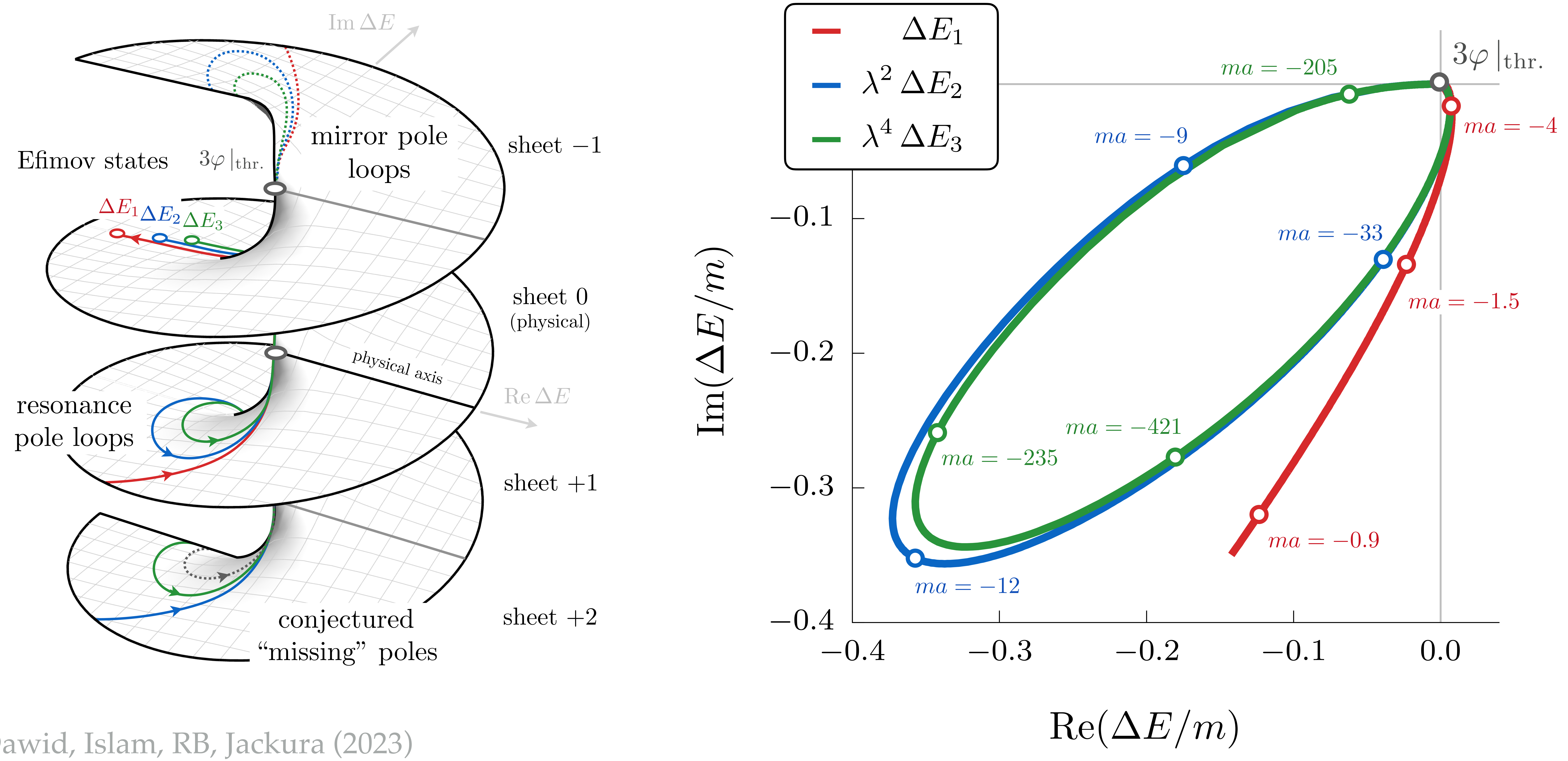


$$i\mathcal{D} = i\mathcal{M}_2 iG i\mathcal{M}_2 + \int_0^{q_{\max}} i\mathcal{M}_2 iG\, i\mathcal{D}$$



Dawid, Islam, RB (2023)

Three-body bound states & resonances



Dawid, Islam, RB, Jackura (2023)

Outstanding milestone: $\pi\rho \leftrightarrow \pi\pi\pi$

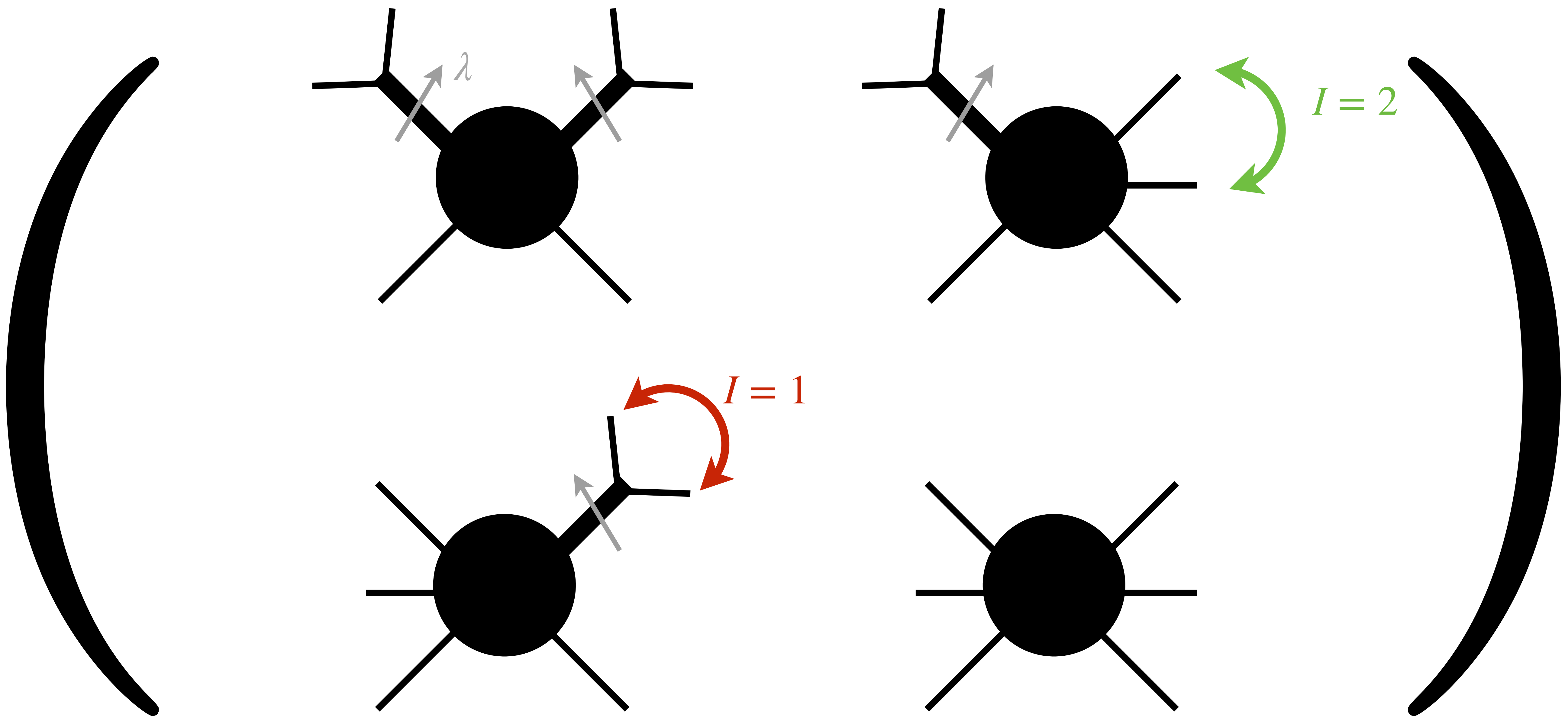
Next obvious place to explore is the $I = 2$ channel, which is not so trivial.

This is a coupled-channel problem: $\left[\, \pi(\pi\pi)_{I=2},\ \pi(\pi\pi)_{I=1} \,\right]$

Progress report:

- ☑ construct operator basis,
- ☑ non-zero angular momentum,
- ☑ two-body resonances with non-zero helicity,
- ☐ get spectrum, [progress]
- ☑ analytic continuing to two-body pole,
- ☑ projecting to cubic irreps,
- ☐ minimizing a matrix of $\mathcal{K}_{\text{df}}$. [outstanding]

Arguably the most challenging lattice QCD calculation to date!



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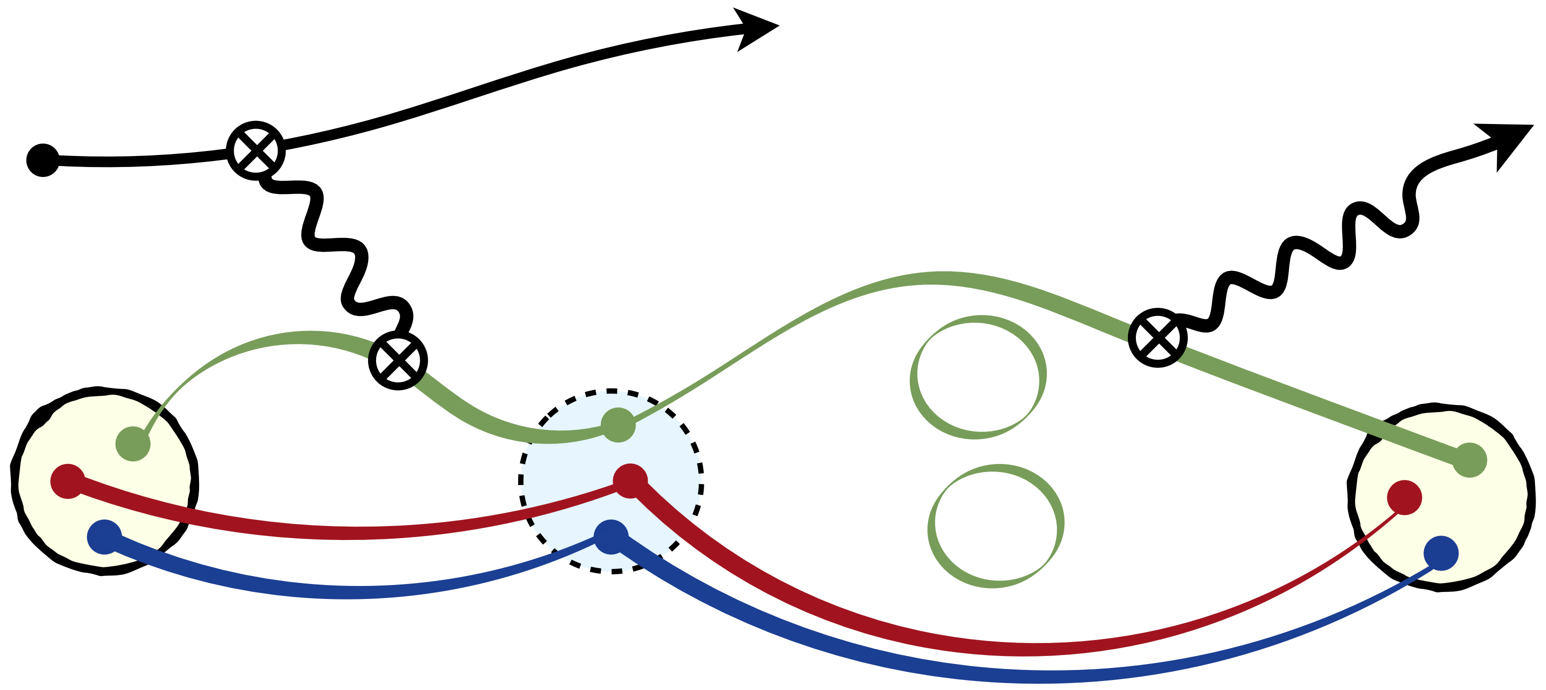
Symbiotic byproducts

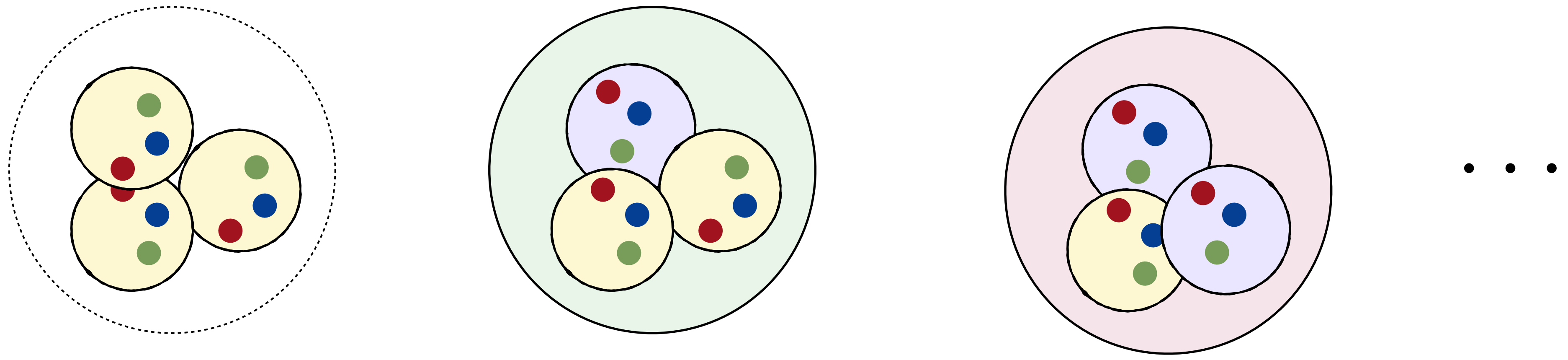
Formal & numerical tools being developed are universal.

These will impact studies in

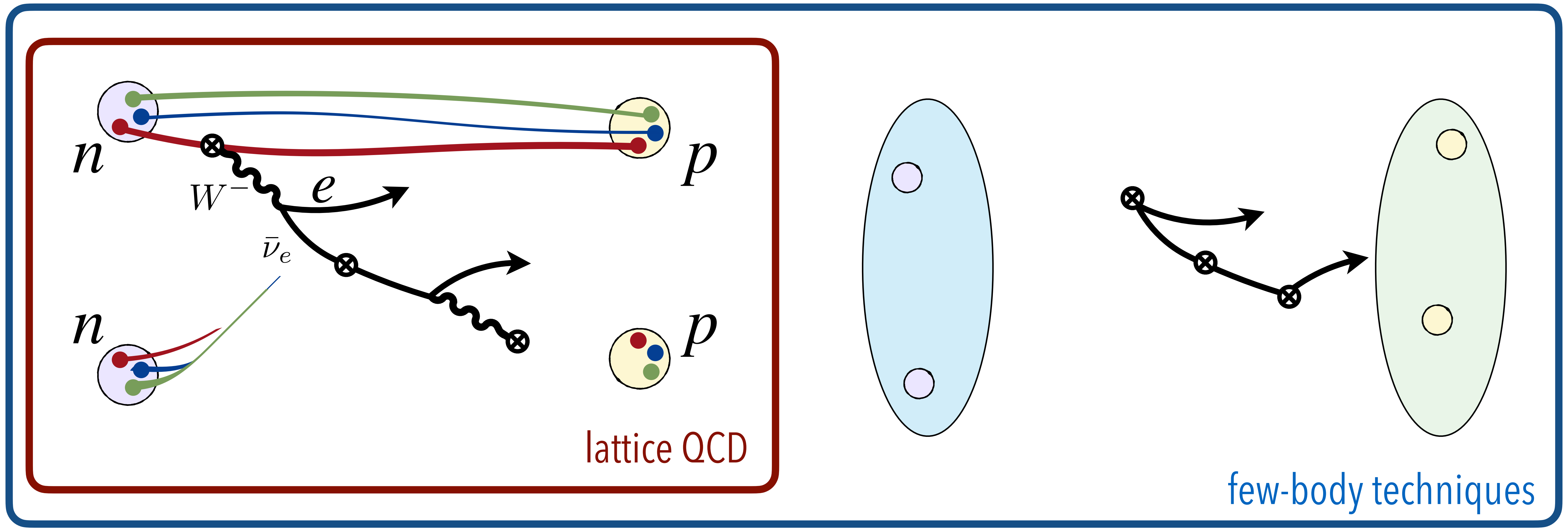
- **hadron structure,**



- **nuclear structure / nuclear-astrophysics,**



- **fundamental symmetries,**



- **universal phenomena,….**

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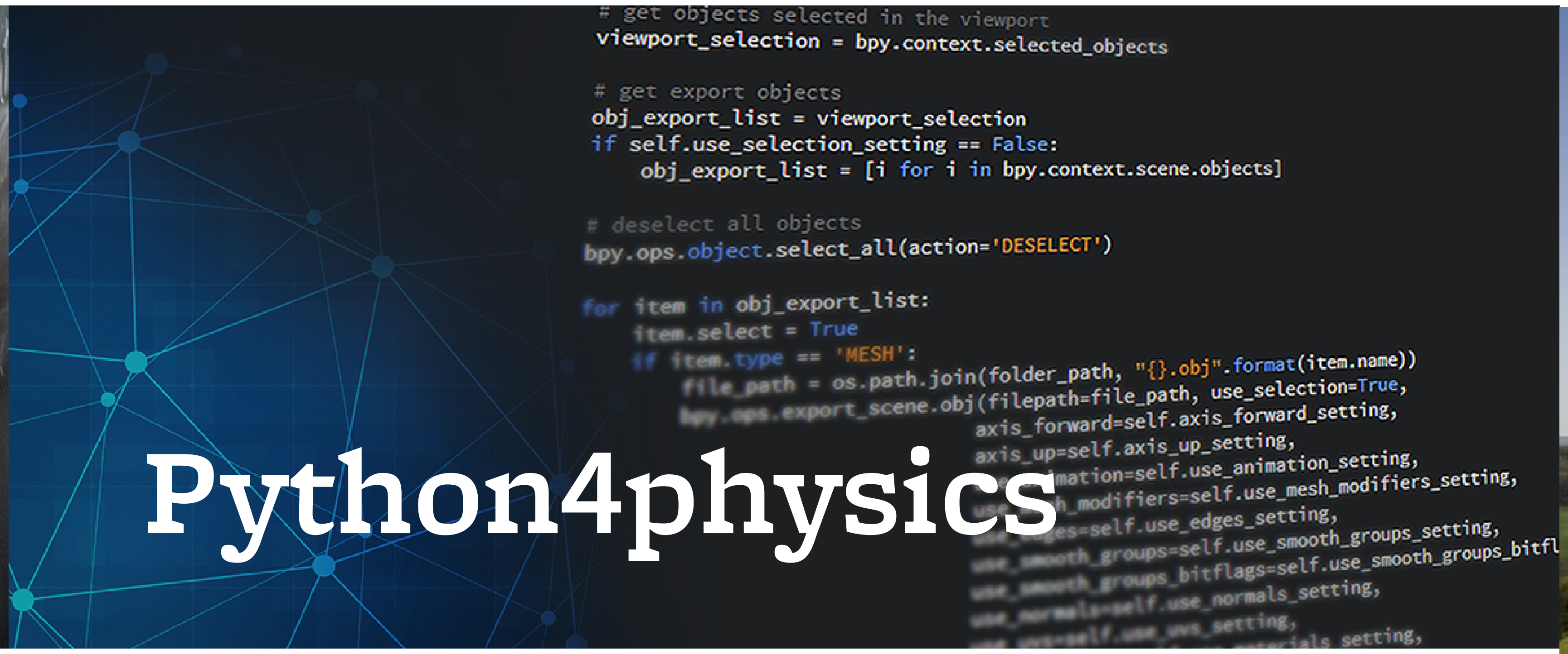

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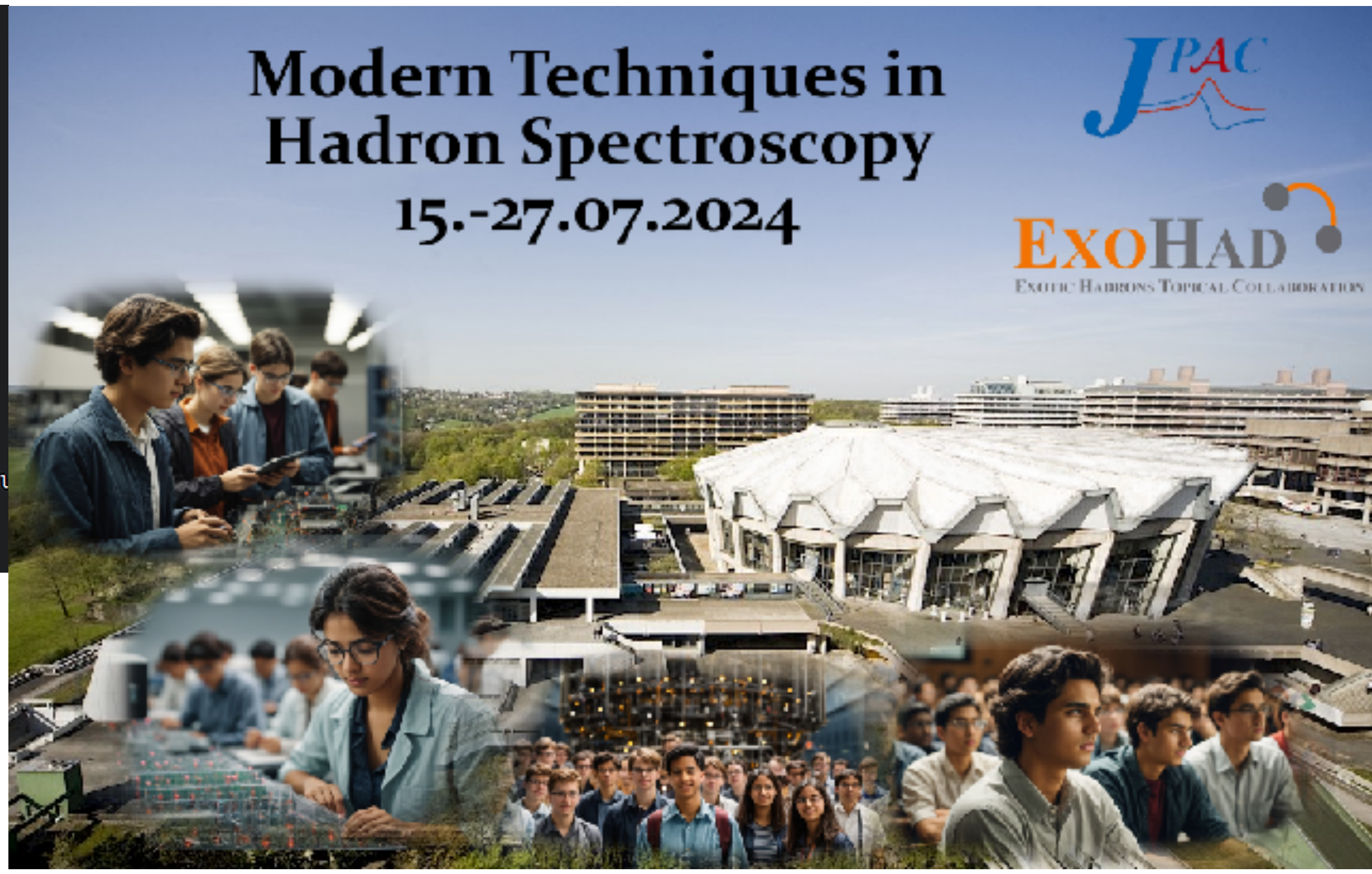

Summer school, 2025

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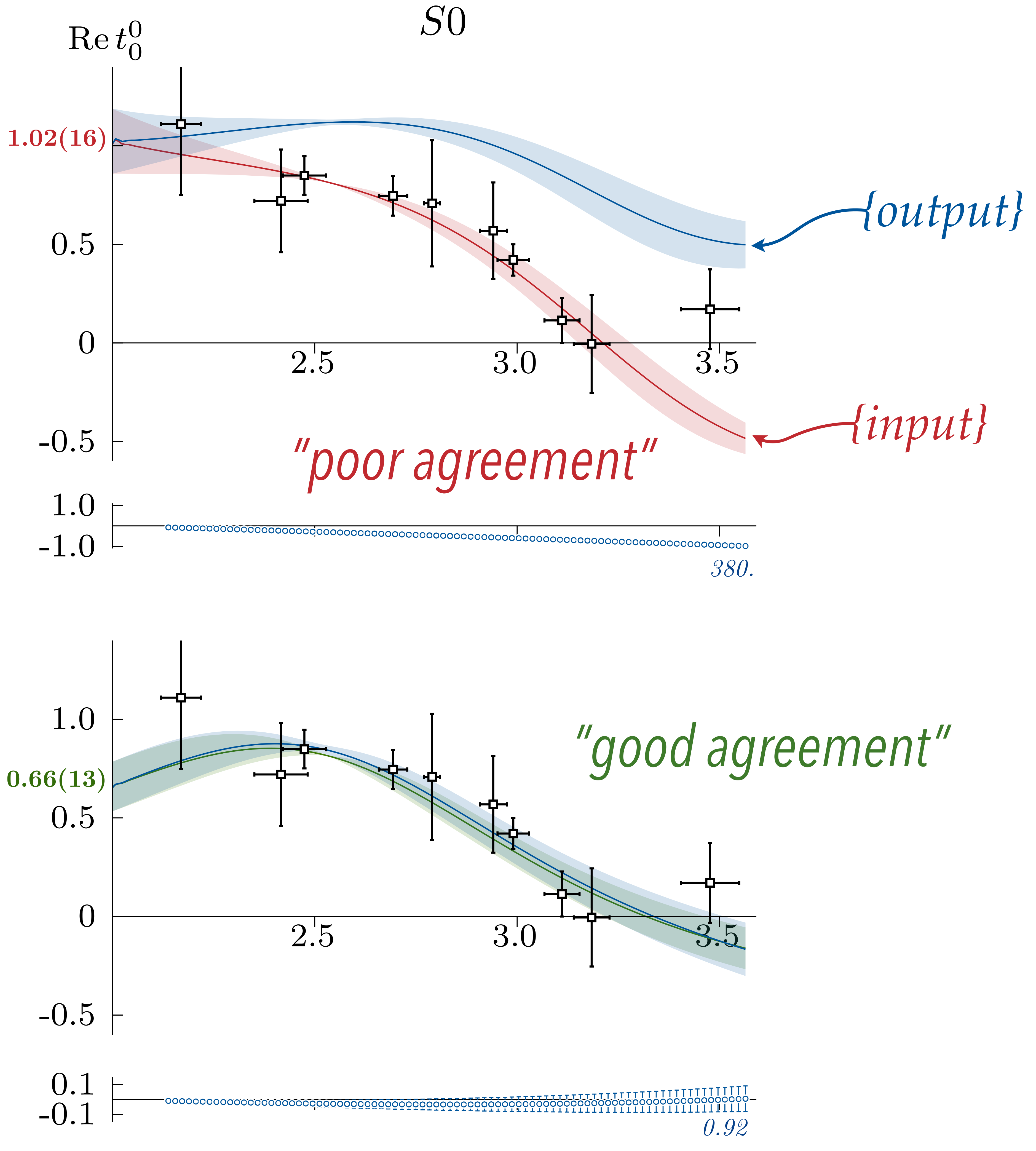
$\pi\pi$ and the σ imposing crossing symmetry



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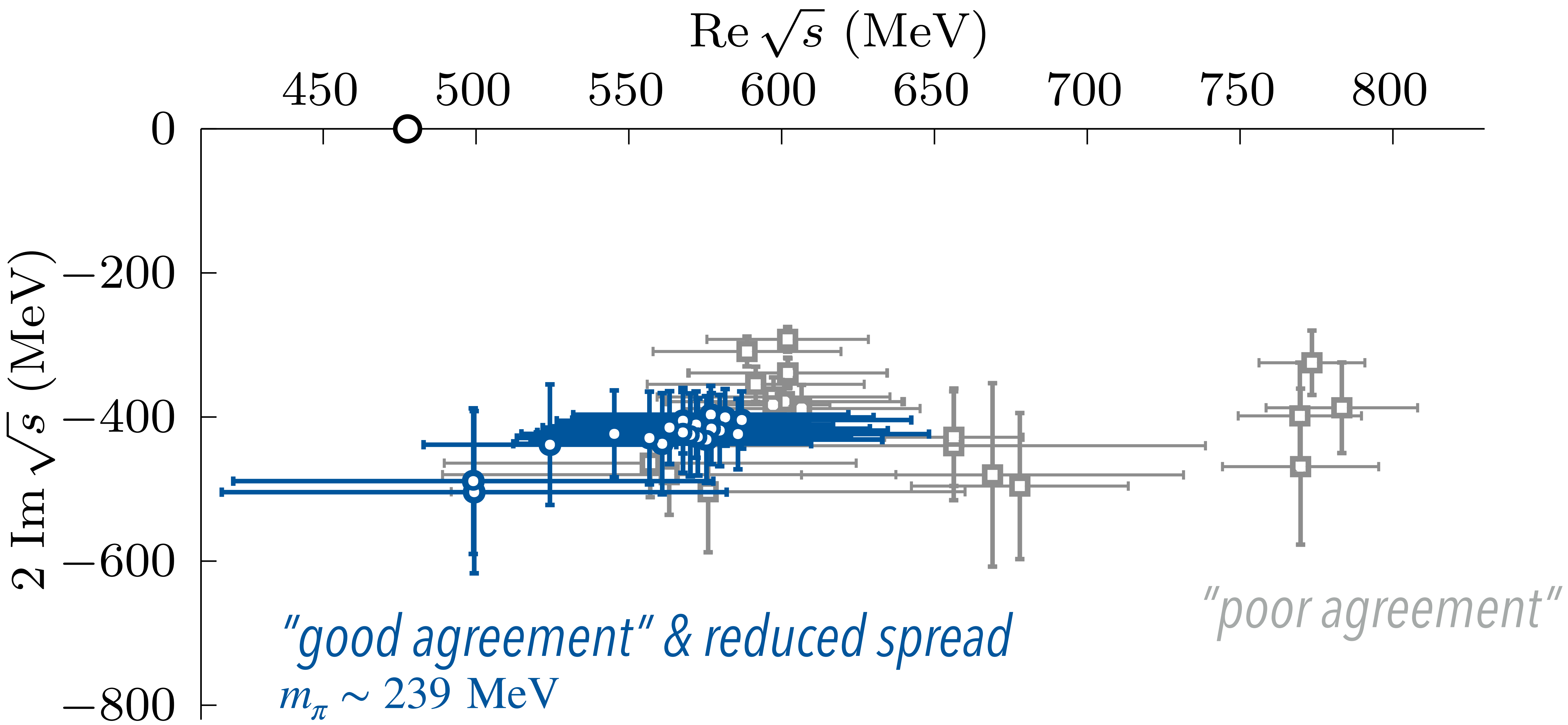

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