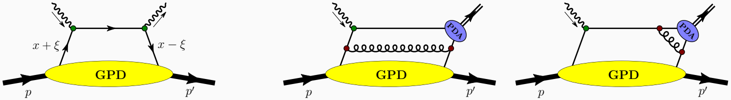
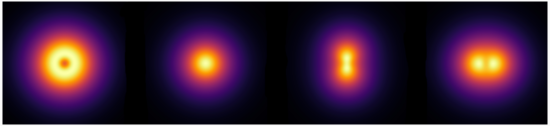
Phenomenology/Global Analysis Highlights and Future Prospects



Ian Cloët
Argonne National Laboratory

[for Global Analysis/Pheno Work Group]



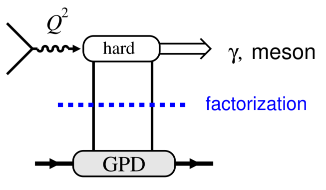
2024 Nuclear Theory Topical Collaborations PI Exchange Meeting

2 May 2024

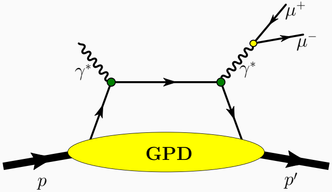
QUARK-GLUON TOMOGRAPHY COLLABORATION

Challenges of Extracting GPDs from Data

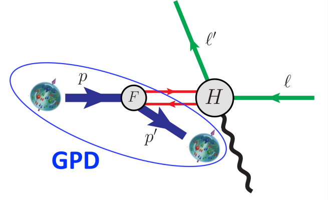
Deeply Virtual Exclusive Processes



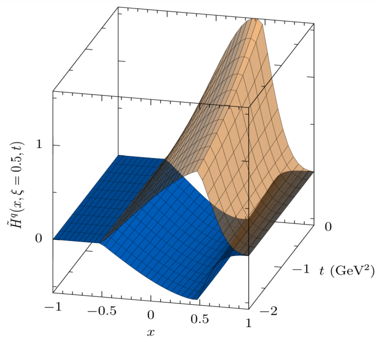
Double DVCS



Single Diffractive Hard EPs



- GPDs encapsulate and generalize many aspects of hadron structure, e.g., em and gravitational form factors, PDFs, DAs, . . .
  - Provides a spacial tomography of hadrons and access to quark & gluon angular momentum (Ji sum rule)
- Many processes are sensitive to GPDs, however, access is not direct and only possible via QCD factorization
  - Need to solve an inverse problem for each process to infer the GPDs
  - These inverse problems are much more challenging than for PDFs and TMDs



Global Analysis & Phenomenology Working Group

- Overarching goal is to bring all the pieces of the QGT collaboration together – Theory, Lattice QCD, and Phenomenology – to perform state-of-the-art Global Analyses of world data to extract GPDs
- Three main efforts/teams:
  - GUMP – GPDs through Universal Moment Parameterization
    Yuxun Guo, Xiangdong Ji, Gabriel Santiago, Kyle Shiells
  - Machine Learning Approach
    Ian Cloët, Christopher Cocuzza, Adam Freese, Leonard Gamberg, Wally Melnitchouk, Andreas Metz, Lillie Mohn, Eric Moffat, Owen Page, Alexei Prokudin, Nobuo Sato, Zhite Yu, and Marco Zaccheddu
  - Nuclear GPDs
    Alberto Accardi and Christian Weiss

- Each team is complementary and together will deliver several approaches for a comprehensive global analysis for nucleon and nuclear GPDs

Beginning with a DVCS database

- Any global analysis is only possible if available data is curated, checked, and put in a consistent format
- Database for QGT is being lead by Alexei Prokudin and Leonard Gamberg at Penn State Berks as an undergraduate project with Lillie Mohn and Owen Page

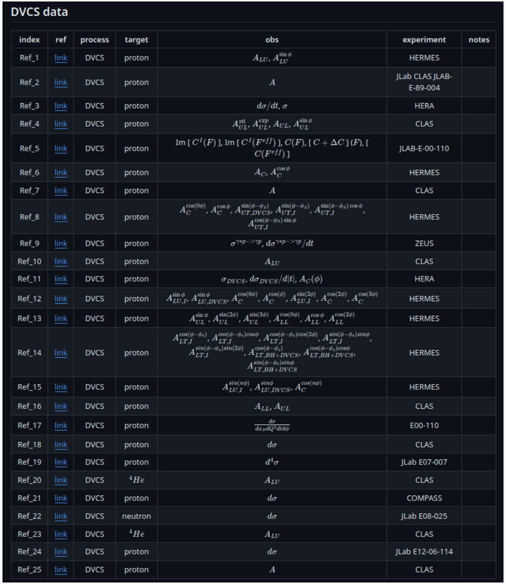
  `https://github.com/prokudin/PSU_PHYS496/tree/master/database`

  - Collected DVCS data from 25 publications, creating 65 excel files
  - Key deliverable for QGT collaboration

Leonard Gamberg

Lillie Mohn

Alexei Prokudin & Owen Page



DVCS data

index	ref	process	target	obs	experiment	notes
Ref_1	link	DVCS	proton	$A_{LU}, A_{LU}^{\sin\phi}$	HERMES	
Ref_2	link	DVCS	proton	A	JLab CLAS JLAB-E-89-004	
Ref_3	link	DVCS	proton	$d\sigma/dt, \sigma$	HERA	
Ref_4	link	DVCS	proton	$A_{UL}^{\pi0}, A_{UL}^{exp}, A_{UL}, A_{UL}^{\sin\phi}$	CLAS	
Ref_5	link	DVCS	proton	$\mathrm{Im}\,[\,C^I(F)\,], \mathrm{Im}\,[\,C^I(F^{eff})\,], C(F), [\,C+\Delta C\,]\,(F), [\,C(F^{eff})\,]$	JLAB-E-00-110	
Ref_6	link	DVCS	proton	$A_C, A_C^{\cos\phi}$	HERMES	
Ref_7	link	DVCS	proton	A	CLAS	
Ref_8	link	DVCS	proton	$A_C^{\cos(0\phi)}, A_C^{\cos\phi}, A_{UT,DVCS}^{\sin(\phi-\phi_S)}, A_{UT,I}^{\sin(\phi-\phi_S)}, A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}, A_{UT,I}^{\cos(\phi-\phi_S)\sin\phi}$	HERMES	
Ref_9	link	DVCS	proton	$\sigma^{\gamma*p->\gamma p}, d\sigma^{\gamma*p->\gamma p}/dt$	ZEUS	
Ref_10	link	DVCS	proton	A_{LU}	CLAS	
Ref_11	link	DVCS	proton	$\sigma_{DVCS}, d\sigma_{DVCS}/d\lvert t\rvert, A_C(\phi)$	HERA	
Ref_12	link	DVCS	proton	$A_{LU,I}^{\sin\phi}, A_{LU,DVCS}^{\sin\phi}, A_C^{\cos(0\phi)}, A_C^{\cos(\phi)}, A_{LU,I}^{\sin(2\phi)}, A_C^{\cos(2\phi)}, A_C^{\cos(3\phi)}$	HERMES	
Ref_13	link	DVCS	proton	$A_{UL}^{\sin\phi}, A_{UL}^{\sin(2\phi)}, A_{UL}^{\sin(3\phi)}, A_{LL}^{\cos(0\phi)}, A_{LL}^{\cos\phi}, A_{LL}^{\cos(2\phi)}$	HERMES	
Ref_14	link	DVCS	proton	$A_{LT,I}^{cos(\phi-\phi_s)}, A_{LT,I}^{cos(\phi-\phi_s)cos\phi}, A_{LT,I}^{cos(\phi-\phi_s)cos(2\phi)}, A_{LT,I}^{sin(\phi-\phi_s)sin\phi}, A_{LT,I}^{sin(\phi-\phi_s)sin(2\phi)}, A_{LT,BH+DVCS}^{cos(\phi-\phi_s)}, A_{LT,BH+DVCS}^{cos(\phi-\phi_s)cos\phi}, A_{LT,BH+DVCS}^{sin(\phi-\phi_s)sin\phi}$	HERMES	
Ref_15	link	DVCS	proton	$A_{LU,I}^{sin(n\phi)}, A_{LU,DVCS}^{sin\phi}, A_C^{cos(n\phi)}$	HERMES	
Ref_16	link	DVCS	proton	A_{LL}, A_{UL}	CLAS	
Ref_17	link	DVCS	proton	$\frac{d\sigma}{dx_BdQ^2dtd\phi}$	E00-110	
Ref_18	link	DVCS	proton	$d\sigma$	CLAS	
Ref_19	link	DVCS	proton	$d^4\sigma$	JLab E07-007	
Ref_20	link	DVCS	^4He	A_{LU}	CLAS	
Ref_21	link	DVCS	proton	$d\sigma$	COMPASS	
Ref_22	link	DVCS	neutron	$d\sigma$	JLab E08-025	
Ref_23	link	DVCS	^4He	A_{LU}	CLAS	
Ref_24	link	DVCS	proton	$d\sigma$	JLab E12-06-114	
Ref_25	link	DVCS	proton	A	CLAS	

QCD Evolution of GPDs

- Fast and reliable code for the QCD evolution of GPDs is crucial for any global analysis of GPD-sensitive data
  - Also important for evolving lattice and model GPD results
- General form of evolution equations:

$$\frac{\mathrm{d}H(x,\xi,Q^2)}{\mathrm{d}\log Q^2} = \int \mathrm{d}y\ K(x,y,\xi,Q^2)\,H(y,\xi,Q^2)$$

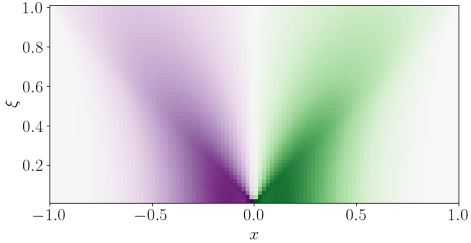
- Solve by discretizing integral and small steps in Q^2

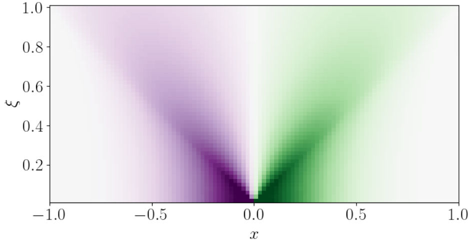
$$\frac{\mathrm{d}H_i(\xi,Q^2)}{\mathrm{d}\log Q^2} = \sum_j K_{ij}(\xi,Q^2)H_j(\xi,Q^2)$$

- Solution is expressed in form of transfer matrices M_{ij} which are independent of initial GPDs

$$H_i(\xi,Q_f^2) = M_{ij}(\xi;Q_i^2 \to Q_f^2)\,H_j(\xi,Q_i^2)$$

- Transfer matrices can be calculated once and then evolution is almost instantaneous


Before evolution $(Q^2 = m_c^2)$

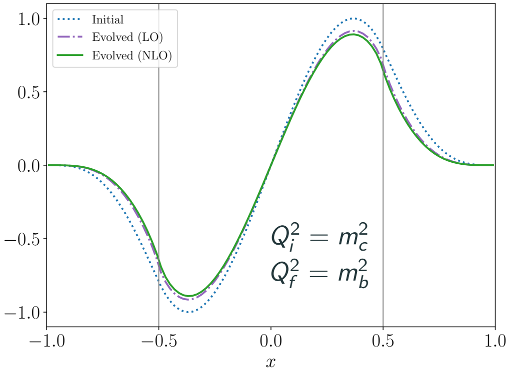

After evolution $(Q^2 = m_b^2)$

Code release and NLO Evolution

- Plan to release LO evolution code with publication within a few months
  - Very limited existing code for community – so this is a crucial milestone for the QGT collaboration
- Also developing NLO evolution code for GPDs
  - To the best of our knowledge no NLO evolution code exists or is publicly available
  - When NLO code is released this will likely be the only NLO GPD evolution code available worldwide
  - Significant milestone for our QGT global analysis efforts
- First results are illustrated for non-singlet NLO evolution
  - In this case, appears to be only a small difference between LO and NLO evolution

Adam Freese

Leading our GPD evolution efforts



Illustrating Challenges using DVCS

- Cross-section for lepto-production process $l(k,\lambda) + A(p,S) \to l(k',\lambda') + \gamma(q',\Lambda') + A(p',S')$ reads

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B\,\mathrm{d}Q^2\,\mathrm{d}|t|\,\mathrm{d}\phi\,\mathrm{d}\varphi} = \frac{\alpha_{\mathrm{em}}^3\,x_B\,y^2}{16\pi^2\,Q^4\sqrt{1+\gamma^2}}\left[|\mathcal{T}_{\mathrm{BH}}|^2 + \mathcal{T}_{\mathrm{I}} + |\mathcal{T}_{\mathrm{DVCS}}|^2\right], \qquad |\mathcal{T}_{\mathrm{DVCS}}|^2 = |\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{UU}} + \dots$$

- At twist-2 in the Compton form factors [Belitsky, Mueller, Kirchner NPB (2002)]:

$$\begin{aligned}|\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{UU}} =& \frac{2\left(2-2y+y^2\right)}{y^2Q^2\left(2-x_B\right)^2}\Big[4\left(1-x_B\right)\left(\mathrm{Re}[\mathcal{H}]^2 + \mathrm{Im}[\mathcal{H}]^2 + \mathrm{Re}\big[\tilde{\mathcal{H}}\big]^2 + \mathrm{Im}\big[\tilde{\mathcal{H}}\big]^2\right)\\ &-x_B^2\left(2\mathrm{Re}[\mathcal{H}]\,\mathrm{Re}[\mathcal{E}] + 2\mathrm{Im}[\mathcal{H}]\,\mathrm{Im}[\mathcal{E}] + 2\mathrm{Re}\big[\tilde{\mathcal{H}}\big]\,\mathrm{Re}\big[\tilde{\mathcal{E}}\big] + 2\mathrm{Im}\big[\tilde{\mathcal{H}}\big]\,\mathrm{Im}\big[\tilde{\mathcal{E}}\big]\right)\\ &\left(x_B^2 + (2-x_B)^2\frac{t}{4M^2}\right)\left(\mathrm{Re}[\mathcal{E}]^2 + \mathrm{Im}[\mathcal{E}]^2\right) - x_B^2\frac{t}{4M^2}\left(\mathrm{Re}\big[\tilde{\mathcal{E}}\big]^2 + \mathrm{Im}\big[\tilde{\mathcal{E}}\big]^2\right)\Big],\end{aligned}$$

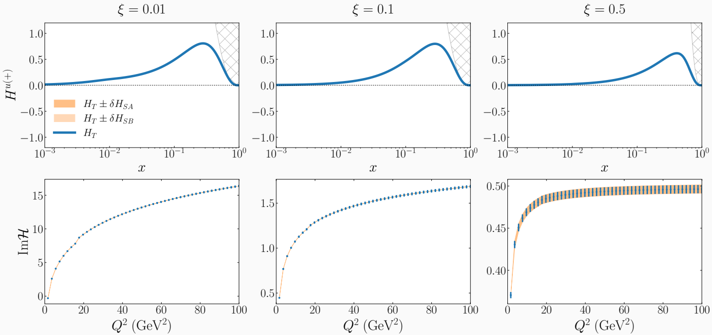
- Each Compton form factor is associated with a GPDs of well-defined twist

$$\mathcal{F}(\xi,t,Q^2) = \int_{-1}^{1}\mathrm{d}x\;C(x,\xi)\;F(x,\xi,t,Q^2), \qquad x \equiv \frac{\bar{k}\cdot n}{\bar{p}\cdot n}, \qquad \xi \equiv \frac{(p-p')\cdot n}{2\,\bar{p}\cdot n},$$

- Note, x is completely integrated out, which gives rise to a challenging inverse problem
  - At twist-2 there are 4 CFFs and at twist-3 there are 12 CFFs

DVCS Inverse Problem has Multiple Solutions

- Multiple solutions first discussed in Bertone, *et al.*, PRD, 114019 (2021)
- These multiple solutions are known as shadow GPDs
  - Represent a significant challenge for extracting GPDs from DVCS data
- Using mock CFF data we studied the ability of QCD evolution to help constrain shadow GPDs

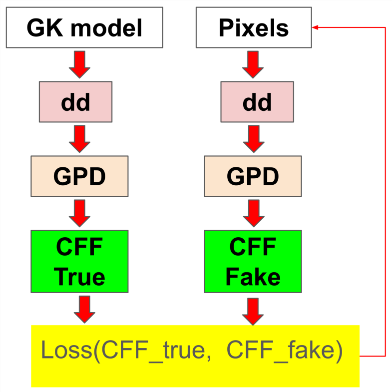

E. Moffat, A. Freese, I. Cloët, *et al.*, PRD **108**, 036027 (2023)

- We find this is possible – over a limited range – with a large lever arm in ξ and Q^2
  - Important caveat – have only considered a very limited class of shadow GPDs
- Points to need for very flexible GPD parametrizations that can capture shadow GPDs
  - Needed for reliable uncertainty quantification of extracted GPDs

Eric Moffat

AI/ML Approach for GPD Extraction

- To ensure polynomiality we work with a double distribution (DD) representation of GPDs
- DDs are represented by millions of pixels which are controlled by a neural network



GK model

dd

GPD

CFF True

Pixels

dd

GPD

CFF Fake

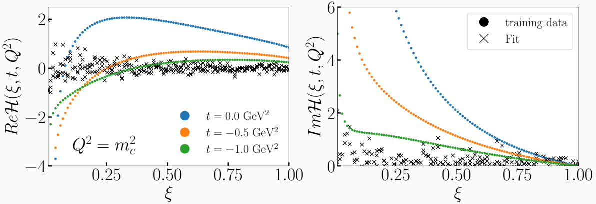
Loss(CFF_true, CFF_fake)

Num pixels (dd space): 2970300

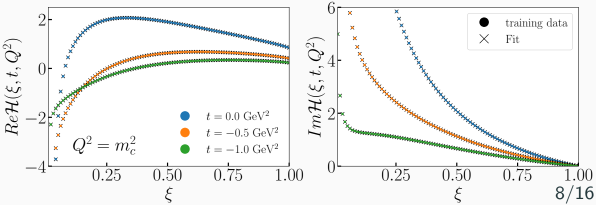
Marco Zaccheddu

Nobuo Sato

Before training



After training (200 epochs)



Uncertainty Quantification and Shadow GPDs

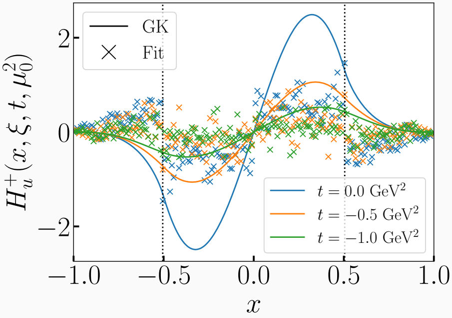
- Pixelized representation of DDs can capture the multiple solutions to the inverse problem between CFFs and GPDs

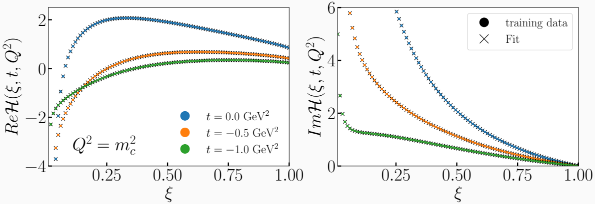
- Each pixelized DD/GPD gives the exact same CFF but can differ substantially from ground truth

- Next steps:
  - Include GPD evolution
  - Add gluons and all quark flavors
  - Carry out uncertainty quantification

- Robust extraction of GPDs from data will likely require data from multiple processes (e.g. DVCS, DVMP, DDVCS, SDHEP, etc.) that are each associated with a different inverse problem

DD is represented by 2,970,300 pixels

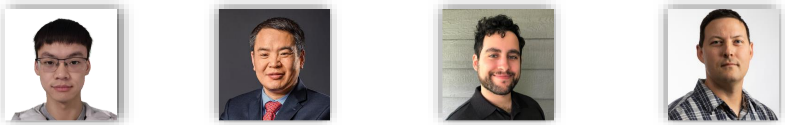




GPDs through Universal Moment Parameterization (GUMP)

Goal: To obtain the state-of-the-art phenomenological Generalized Parton Distributions (GPDs) through global analysis of both experimental data and lattice QCD simulations, utilizing a *universal moment parameterization* method.

Collaborators:



Yuxun Guo (Postdoc)
Lawrence Berkeley Lab.

Xiangdong Ji (PI)
University of Maryland

M. Gabriel Santiago (Postdoc)
Center for Nuclear Femtography

Kyle Shiells (PI)
University of Manitoba

GPDs in terms of Moments

GPDs can be formally expanded in the conformal moment space:

D. Mueller and A. Schafer 2006

$$F(x,\xi,t)=\sum_{j=0}^{\infty}(-1)^j p_j(x,\xi)\mathcal{F}_j(\xi,t)$$

$p_j(x,\xi)$: Orthogonal basis in terms of Gegenbauer polynomials

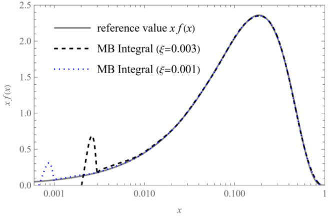
$\mathcal{F}_j(\xi,t)$: Moments of GPDs to be parameterized

Advantage: important constraints like the polynomiality condition can be put in easily.

Whereas GPDs in x-space can be reconstructed by resumming all the moments through a complex integral in the moment space.

$$F(x,\xi,t)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dj\,\frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)\ ,$$

Example of the reconstruction of GPD in x-space with its moments



Universal Parameterization of GPD Moments

Moments of GPDs are expandable in ξ due to the polynomiality condition. For small $\xi \lesssim 0.3$ which covers most of the current data, we consider the expansion of moments

$$\mathcal{F}_j(\xi,t)=\mathcal{F}_{j,0}(t)+\xi^2\mathcal{F}_{j,2}(t)+\xi^4\mathcal{F}_{j,4}(t)+\cdots$$

The first term describes GPDs at $\xi=0$, and is parameterized with a 5-parameter ansatz (N,α,β,α',b):

$$\mathcal{F}_{j,0}(t)=NB(j+1-\alpha,1+\beta)\frac{j+1-\alpha}{j+1-\alpha(t)}\beta(t) \qquad \beta(t)=e^{-b|t|}$$

Euler Beta Function

Regge trajectory $\alpha(t)=\alpha+\alpha' t$

- Beta function $B(j+1-\alpha,1+\beta)$: corresponds to the PDF ansatz $x^{-\alpha}(1-x)^{\beta}$ in forward limit
- Regge trajectory: modify the small-x behavior at different t in the form of $x^{-\alpha(t)}$
- The residual term $\beta(t)$: motivated by the measured t-dependence in elastic scattering.

The ξ-dependence of GPD can in principle be independently parameterized. Here we instead parameterize them with simple ratios: $\mathcal{F}_{j,2}(t)=R_{\xi^2}\mathcal{F}_{j,0}(t)$ $\mathcal{F}_{j,4}(t)=R_{\xi^4}\mathcal{F}_{j,0}(t)$ to avoid unconstrained parameters due to the lack of input.

Strategy for the global analysis

Experimental data and constraints

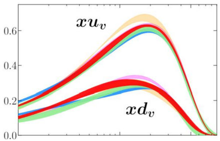
- ❑ Polarized and unpolarized PDFs from global analysis
  - Alternatively, one can fit to (polarized) DIS directly
- ❑ Neutron/ Proton charge form factors from global analysis
- ❑ Deeply virtual Compton scattering data at JLab/HERA
- ❑ Deeply virtual meson productions data at HERA

Lattice QCD simulations

- ❑ Lattice simulations of nucleon generalized form factors
- ❑ Lattice simulations of unpolarized and helicity GPDs at zero and non-zero ξ (skewness)

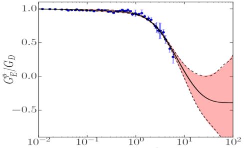
Sequential fit as first step to accelerate the convergence

Example of global PDF fit



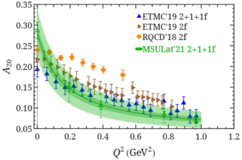
JAM 2022

Example of charge form factor fit



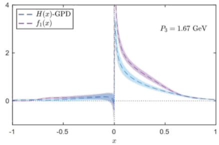
Q^2 [GeV^2] Z. Ye et al 2018

Example of lattice form factors



Huey-Wen Lin 2022

Example of lattice GPD



C. Alexandrou et. al. 2020

Semi-forward

H → E → $\widetilde{H}$ → $\tilde{E}$

Off-forward

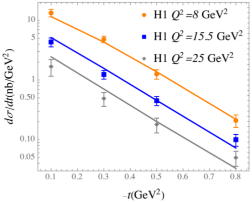
- JAM (2022) PDF global analysis results
- Globally extracted electromagnetic form factors (*Z. Ye et al* 2018)
- Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)

DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)

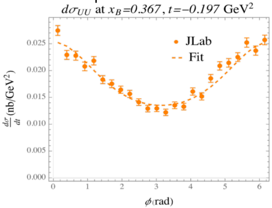
(Preliminary) examples of GUMP fits

Quark GPDs: first extraction of quark GPDs with lattice input:

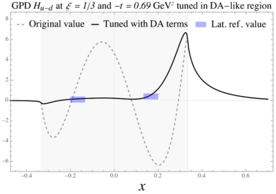
Example of fit to DVCS at H1



Example of fit to DVCS at JLab

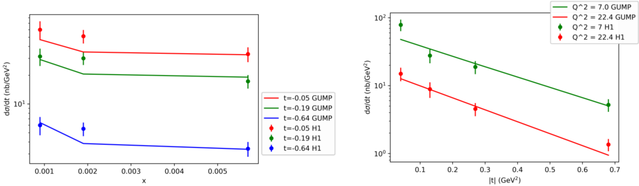


Example of GPDs tuned to lattice input



(Preliminary) Gluon GPDs: first GPD analysis of DV J/ψP at NLO:

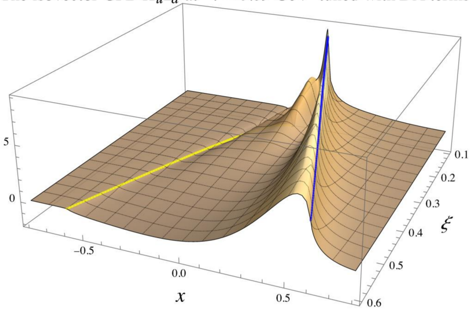
Preliminary fit to the H1 measurement of DV J/ψP



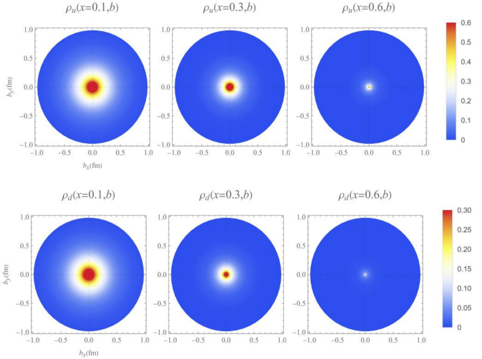
The GUMP extraction of GPDs

Extracted quark GPDs tuned with lattice input:

The isovector GPD H_{u-d} at $-t = 0.69\ \mathrm{GeV}^2$ tuned with DA terms



Nucleon tomography with extracted quark GPDs



Future development: i). More meson productions to be included.

ii). Systematic implementation of NLO calculations in the analysis.

iii). Simultaneous extraction of quark and gluon GPDs.

Summary and Outlook

- A key question we would like to answer is: At what resolution can we extract an image of the proton?
- Pixelization with a smoothing/cooling algorithm is one pathway to address this question
- Excellent progress on the milestones from all teams
- We are building a US lead effort to extract GPDs from data
- Other GPD global analysis efforts exist:
  - Gepard [`https://gepard.phy.hr/`]. Contact person is Krešimir Kumerički.
  - PARTONS [`https://partons.cea.fr`]. Contact person is Hervé Moutarde.
  - EXCLAIM. Contact person is Simonetta Liuti.