Radiative corrections to β decay in Effective Field Theory

2024 Topical Collaboration Principal Investigators' Exchange Meeting
May 2nd, 2024

EFT objectives and key steps

YEAR 1: Primary objectives and key steps leading to them

BETA-2 Develop EFT formalism for A=2 systems to $O(G_F\alpha)$

Y1 Compute $A = 2$ weak transitions in pionless EFT and chiral EFT to $O(G_F\alpha)$, including sub-leading corrections in the chiral counting in Q/Λ. [LANL,UW]

Y1 Identify the two-body transitions operators that need to be included in consistent many-body calculations to a given order. [LANL,UW]

YEAR 3: Primary objectives and key steps leading to them

BETA-2 EFT analysis of radiative corrections to few-body systems

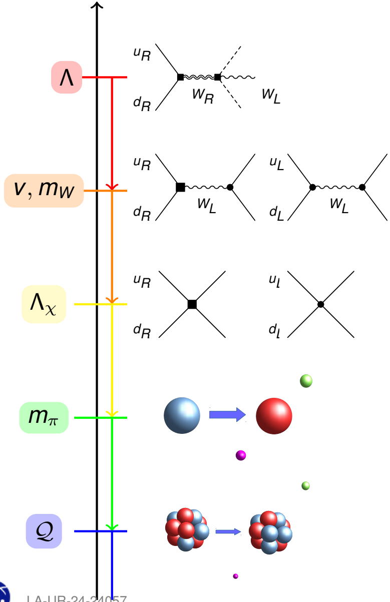
Y2 EFTs analysis of radiative corrections to *pp* fusion. [LANL, UTK, UW]

Y3 EFTs analysis of radiative corrections to muon capture on deuterium. [LANL, UTK, UW]

BETA-3 Calculation of δ_C, δ_{NS} in low-A systems with various methods – benchmarking

Y3 Calculation of δ_C, δ_{NS} corrections with QMC methods. [ANL, LANL & WUSTL]

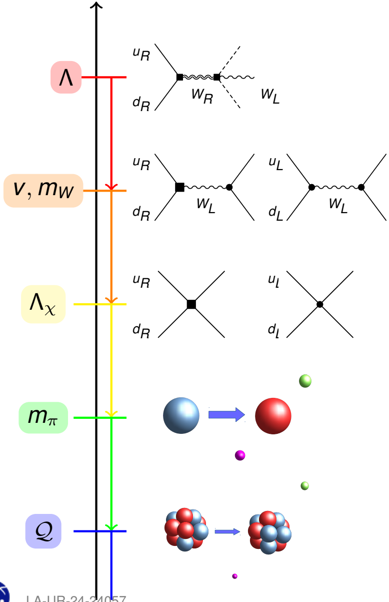
Effective Field Theory framework



- Exploit the hierarchy of scales in β decay and organize decay rate in an expansion in

$$\alpha,\ \alpha_s,\ \epsilon_\chi = \frac{m_\pi}{\Lambda_\chi},\ \epsilon_{\not{\pi}} = \frac{\mathcal{Q}}{m_\pi}$$

Effective Field Theory framework



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- in perturbative regime, state of the art EFT tools to resum $\log \Lambda_\chi / m_W$ at higher accuracy

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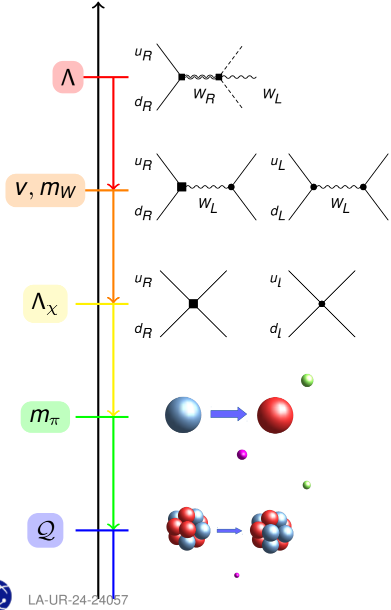

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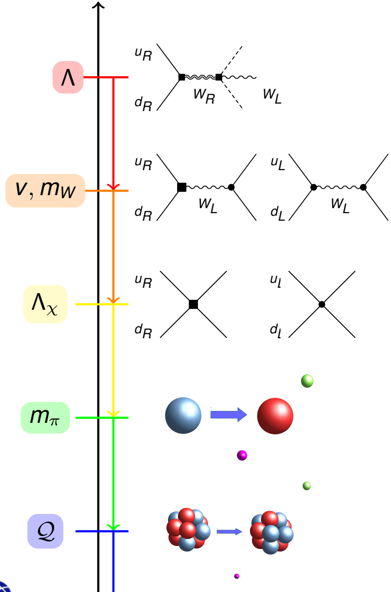

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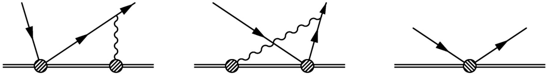
- in perturbative regime, state of the art EFT tools to resum $\log \Lambda_\chi / m_W$ at higher accuracy
- low energy EFTs to factorize photons with different virtuality
- hard: $(q_\gamma^0, \vec{q}_\gamma) \sim \Lambda_\chi$. Need input from LQCD
- potential $(q_\gamma^0, \vec{q}_\gamma) \sim (0, m_\pi)$ and soft $(q_\gamma^0, \vec{q}_\gamma) \sim (m_\pi, m_\pi)$. Derive transition operators for *ab initio* calculations
- ultrasoft: $(q_\gamma^0, \vec{q}_\gamma) \sim \mathcal{Q}$. Long distance dynamics, see nucleus as a whole

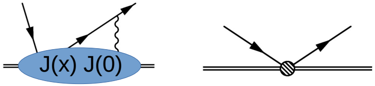
EFT for neutron decay

V. Cirigliano, W. Dekens, EM, O. Tomalak, PRD 108 (2023) 5, 053003

χPT



Fermi theory + QED +QCD



1. From m_W to Λ_χ

- correct anomalous dimension at $\mathcal{O}(\alpha^2)$ ✓

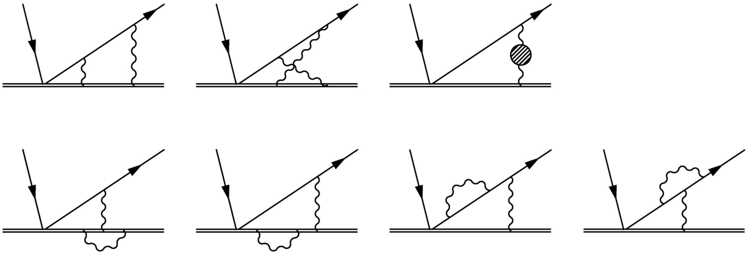
2. From quarks to nucleons

- representation of χPT $\mathcal{O}(\alpha)$ LEC in terms of hadronic objects with full tracking of the scale and scheme dependence ✓

$$g_V(\mu_\chi) = \overline{C}^r_\beta(\mu)\left[1 + \overline{\Box}^V_{\rm Had}(\mu_0) - \frac{\alpha(\mu_\chi)}{2\pi}\left(\frac{5}{8} + \frac{3}{4}\ln\frac{\mu_\chi^2}{\mu_0^2} + \left(1 - \frac{\alpha_s}{4\pi}\right)\ln\frac{\mu_0^2}{\mu^2}\right)\right]$$

$$\overline{\Box}^V_{\rm Had}(\mu_0) = -e^2\int\frac{i{\rm d}^4q}{(2\pi)^4}\frac{\nu^2+Q^2}{Q^4}\left[\frac{T_3(\nu,Q^2)}{2m_N\nu} - \frac{2}{3}\frac{1}{Q^2+\mu_0^2}\left(1 - \frac{\alpha_s(\mu_0^2)}{\pi}\right)\right]$$

EFT for neutron decay



3. From Λ_χ to m_e

- consistent NLL resummation by identifying $\mathcal{O}(\alpha^2)$ anomalous dimension from HQET literature

X. D. Ji and M. Ramsey-Musolf, ‘91 ; V. Gimenez, ‘92; D. J. Broadhurst and A. G. Grozin, ‘99

- & removing unphysical scale dependencies in the “Fermi function”

EFT for neutron decay

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3}(1+3\lambda^2)\cdot f_0 \cdot (1+\Delta_f)\cdot(1+\Delta_R), \qquad \lambda = g_A/g_V$$

- new prescription for the phase space corrections
  captures $\mathcal{O}((\alpha\pi)^n)$ corrections and accurate up to $\mathcal{O}(\alpha^2)$

$$\Delta_f = 3.573(5)\%$$

- radiative corrections with NLL resummation of *all* large logs

$$\Delta_R = 4.044(27)\%$$

error dominated by nonperturbative $\overline{\square}^V_{\rm Had}(\mu_0)$

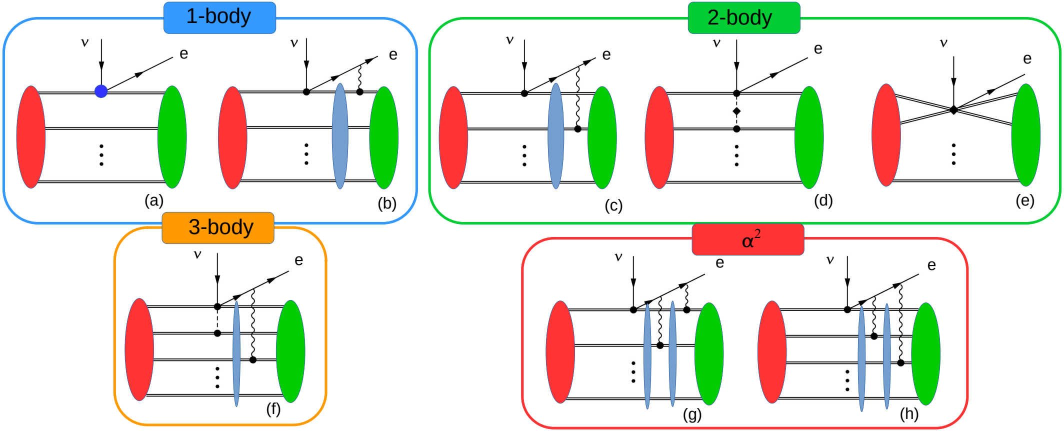
- using the most precise experimental input on neutron lifetime and g_A
  (τ_n from UCNτ and λ from PERKEO-III)

$$V_{ud}^{\text{n, best}} = 0.97402(2)_{\Delta_f}(13)_{\Delta_R}(35)_{\lambda}(20)_{\tau_n}[42]_{\text{total}},$$

approaching superallowed precision

EFTs for nuclear decays

V. Cirigliano, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter, EM, `arXiv:2405.xxxxx`



- diagrams as (*c*) and (*f*) sensitive to potential and soft photon modes, not present in 1-nucleon case

EFTs for nuclear decays

1. proved an EFT factorization formula

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{2(G_F V_{ud})^2}{(2\pi)^5} W(E_e, \mathbf{p}_e\, \mathbf{p}_\nu)\, \tilde{C}(E_e)\, \bar{F}(\beta, \mu_\chi)\, \left[1+\tilde{\delta}'_R(E_e, \mu_\chi)\right](1-\bar{\delta}_C)\, \left[1+\tilde{\delta}_{\mathrm{NS}}(E_e, \mu_\chi)\right] \left[C_{\mathrm{eff}}^{(g_V)}(\mu_\chi)\right]^2,$$

- μ_χ acts as a renormalization/factorization scale, separating contribs. from different virtualities
- $C_{\mathrm{eff}}^{(g_V)}$ encodes contributions from hard photons (g_V) and soft photons down to $\mu_\chi \sim m_\pi$
- δ_{NS} ("nuclear structure dependent corrections") and δ_C ("Coulomb corrections") can be understood as arising from potential modes in chiral EFT
- $\bar{F}$ ("Fermi function") and δ'_R ("Sirlin function" + $\mathcal{O}(\alpha^2)$) capture long distance, ultrasoft photon exchanges

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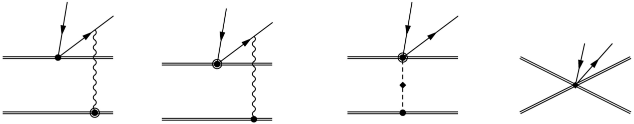
2. resummation of $\log m_e/m_\pi$

- derived anomalous dimension of $C_{\rm eff}^{(g_V)}$ at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha^2 Z(Z+1))$
- showed that the scale dependence $C_{\rm eff}^{(g_V)}$ matches log in the Sirlin function, and "nuclear radius" log in Fermi function

δ_{NS} in chiral EFT



3. transition operators for δ_{NS} at $\mathcal{O}(G_F \alpha \epsilon_\chi)$ and $\mathcal{O}(G_F \alpha^2)$

$$\mathcal{V}^0 = \mathcal{V}_0^{\rm mag} + \mathcal{V}_0^{\rm rec} + \mathcal{V}_0^{\rm CT}, \qquad \delta_{\rm NS} = \frac{2}{M_F} \langle f | \mathcal{V}_0 | i \rangle$$

- long distance part very close to classical formalism by J. Towner and I. Hardy

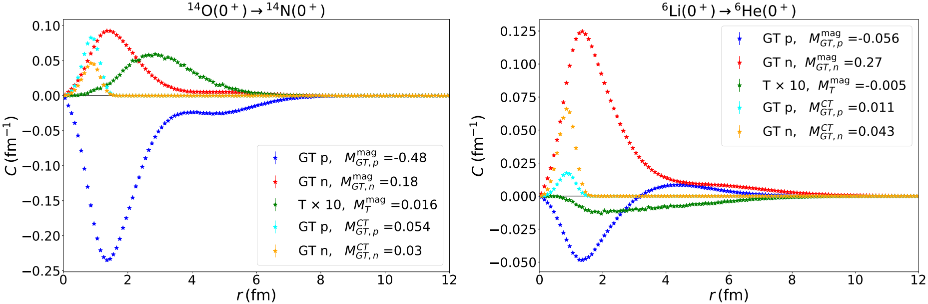
$$\mathcal{V}_0^{\rm mag}(\mathbf{q}) = \sum_{j<k} \frac{e^2}{3} \frac{g_A}{m_N} \frac{1}{\mathbf{q}^2} \left(\boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\sigma}^{(k)} + \frac{1}{2} S^{(jk)} \right) \left[(1+\kappa_p) \tau^{+(j)} P_p^{(k)} + \kappa_n \tau^{+(j)} P_n^{(k)} + (j \leftrightarrow k) \right]$$

- need contact interactions at $\mathcal{O}(G_F \alpha \epsilon_\chi)$

$$\mathcal{V}_0^{\rm CT} = e^2 \left(g_{V1}^{NN} O_1 + g_{V2}^{NN} O_2 \right),$$

need to be fitted jointly to V_{ud}, or matched to LQCD/dispersion theory

First ab initio calculation of δ_{NS} on ^{14}O



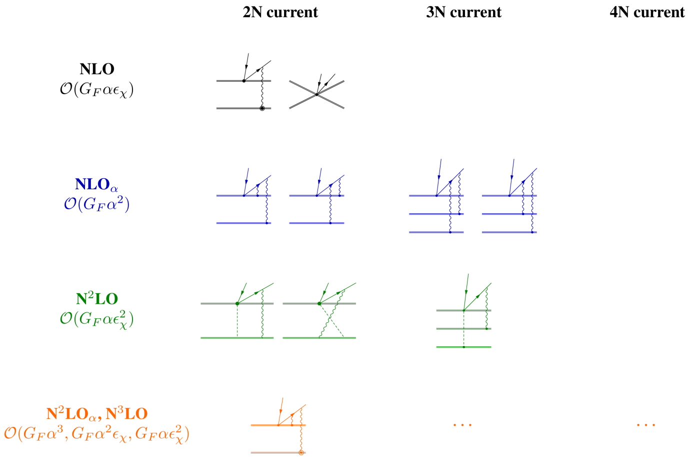
- calculated ME of $\mathcal{V}_0$ in $A = 6$ and $A = 14$ with Variational and Auxiliary Field Diffusion Monte Carlo

$$\delta_{NS}(^{14}O)\Big|_{\mathrm{EFT+QMC}} = -(1.76 \pm 0.88_{\mathrm{LEC}} \pm 0.35_{\mathrm{trunc}}) \cdot 10^{-3} \qquad \delta_{NS}(^{14}O)\Big|_{\mathrm{shell\ model}} = -(1.96 \pm 0.50) \cdot 10^{-3}$$

first *ab initio* calculation of δ_{NS}!

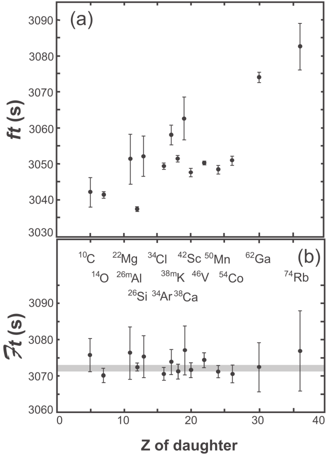
- we treat short-distance operators as an error
- need full theory uncertainty quantification (vary Hamiltonian, cut-off, *ab initio* method) to go beyond uncontrolled shell-model error

Next steps



- derive transition operators at higher orders, to test convergence and theory errors
- uncertainty quantification for δ_{NS} and δ_C

Next steps



from J. Hardy and I. Towner
PRC 102, 045501 (2020)

- establish framework for joint fits to V_{ud}, g_{V1}^{NN} and g_{V2}^{NN}
- need *ab initio* methods with *same* nuclear interactions across the chart

NTNP Coll. has all the expertise to do this!

- collaboration with experimentalists to motivate higher precision in light nuclei (^{10}C, ^{18}Ne)
- and to carry out global fit

Conclusion

- achieved objectives for Y1, and on track on Y2

  including *pp* fusion and few-body processes
- making fast progress on the QMC calculations of δ_{NS} in light nuclei

- EFT framework can systematize calculations of radiative corrections to β decays
- provides the glue between LQCD, dispersive and *ab initio* methods
  in effort towards controlled theory errors

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- using the PDG input

$$V_{ud}^{\text{n, PDG}} = 0.97430(2)_{\Delta_f}(13)_{\Delta_R}(82)_{\lambda}(28)_{\tau_n}[88]_{\text{total}}.$$