# Quark-Gluon Tomography Collaboration: Lattice Highlights and Future Prospects

David Richards (JLab),
for *Lattice Working Group*

# The Big Picture


**Post-docs (PD):**

**ANL:** ***Xiang Gao***
**BNL*****: Raza Sufian***
**JLab:** ***Joe Karpie,***
***CNF: Fatma Aslan***
**UKY:** ***Bigeng Wang***
**WM:** ***Herve Dutrieux***

***Cross-pollination***

**Graduate Students (GS):**

**UMD:** ***Yushan Su, Jinchen He***
**Temple:** ***Joey Delmar, Josh Miller, Joey Torciello***
**WM:** ***Chris Chamness, Daniel Kovner, Alex Sturzu***

**Year 1: L1.1 Pion DA and connection to DVMP - ANL (Yong Zhao)**

**Year 2: L2.1 Framework for GPDs within short-distance factorization - WM (Kostas Orginos)**

**Year 3: L3.1 Calculation of Quark Spin and Trace Anomaly - UKY (Keh-fei Liu)**
**L3.2 Calculation of Pion GPDs - SUNY (Sergei Syritsyn)**

**Year 4: L4.1 Nucleon twist-2 GPDs with controlled $m_\pi$ uncertainty - JLab (David Richards)**
**L4.2 Framework and evaluation of twist-3 GPDs and quark AM - Temple (Martha Constantinou)**
**L4.3 GFF at physical $m_\pi$ with controlled finite vol and excited state - MIT (Phiala Shanahan)**

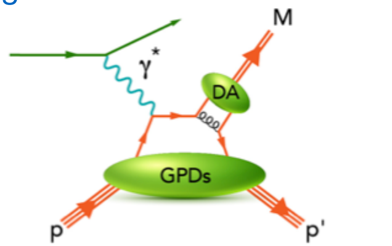
**Year 5: L5.1 Implementation of lattice FFs and GFFs in global amplitude analysis - MIT (Will Detmold)**
**L5.2 Framework/implementation of LQCD twist-2/twist-3 GPDs in global analysis - WM (Chris Monahan)**

*Milestones are not a straightjacket, but what we* ***must*** *do………..*

# Pion Distribution Amplitude

Pion Distribution Amplitude describes internal structure in Exclusive Processes, e.g. Electromagnetic Form Factors, at high momentum Transfers; DVMP

**First year lattice milestone**




$$\langle 0|\bar{d}(z_2 n)\gamma\cdot n\gamma_5 u(z_1 n)|\pi(p)\rangle = i f_\pi (p\cdot n)\int_0^1 dx\, e^{-i(z_1 x + z_2(1-x))p\cdot n}\phi_\pi(x,\mu^2)$$

*Spatially* separated

LaMET or Quasi-PDF

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

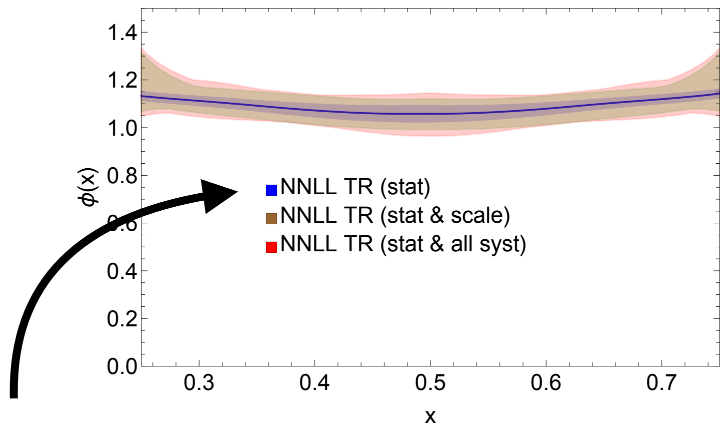
$$\langle 0|\bar{d}(-z/2)\gamma_\rho\gamma_5 u(z/2)|\pi(p)\rangle$$

SDF or Pseudo-PDF

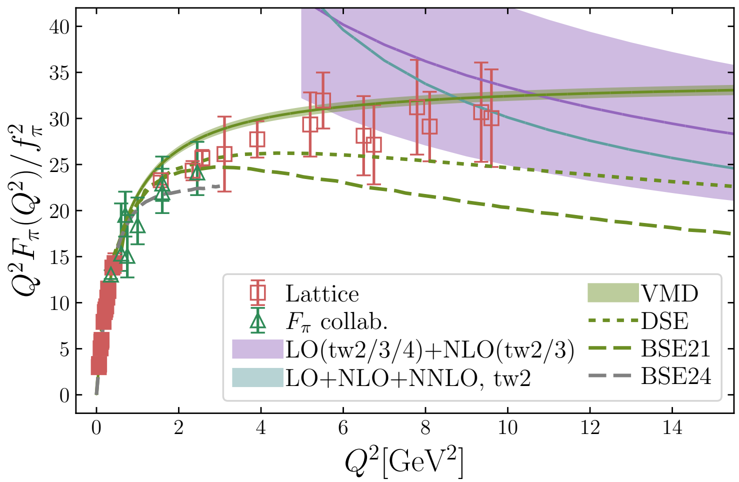
A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

*LaMET calculation with chiral fermions and physical pion mass on a fine lattice* R.Zhang, Y.Zhao *et al*. (in preparation)

JLab@12 GeV and EIC




NLL Threshold resumption




See also D.Kovner et al., arXiv:2401.06858

H.T.Ding *et al*., arXiv:2404.04412



Phys.Lett.B 852 (2024) 138617

# Framework for GPDs (L2.1 + most everything)

Pseudo-PDF approach

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Ioffe time $\nu = p \, . \, z$

Computed on lattice | Perturbatively calculable | Ioffe-time Distribution

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$
$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

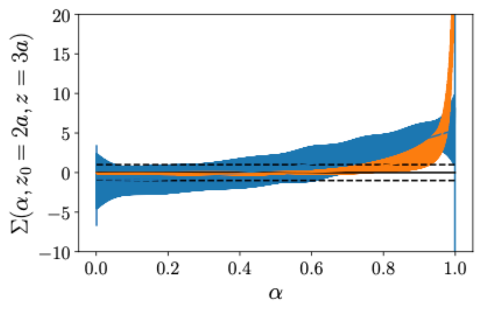
Need data for all v, *or additional physics input*

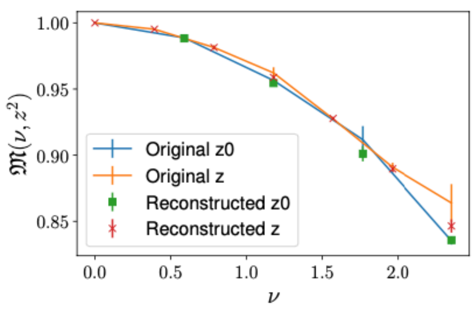
H.Dutrieux et al., J. High Energy Phys. 2024, 61 (2024)

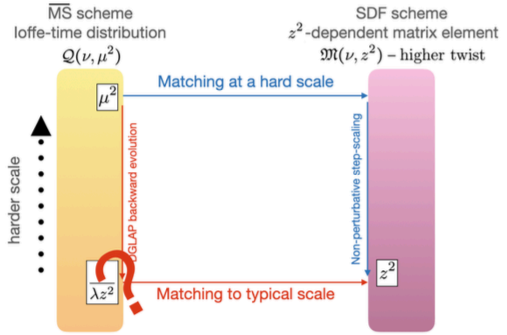
Explore the non-perturbative evolution of non-singlet PDF within short-distance factorization approach.

Introduce step-scaling function

$$\mathcal{M}(\nu; z_1) = \int_0^1 d\alpha\, \Sigma(\alpha; z_0^2, z_1^2) \mathcal{M}(\alpha\nu; z_0^2)$$



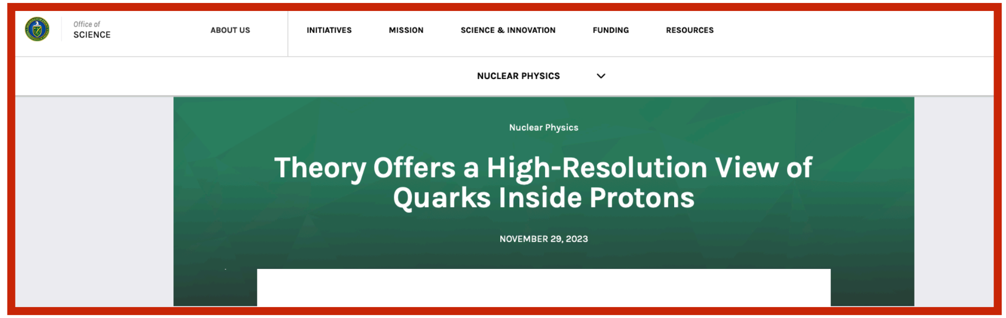

Calculations in small volume + step scaling: *increased range of $\nu$*

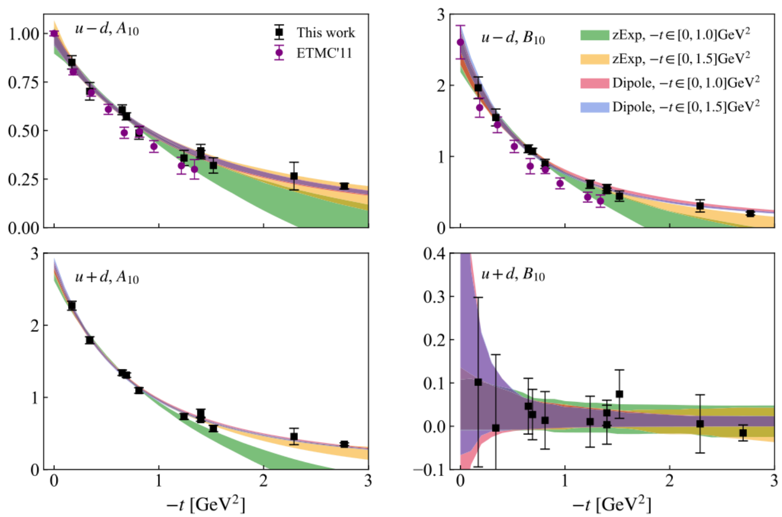
*Example extraction of step-scaling function using Bayesian reconstruction*

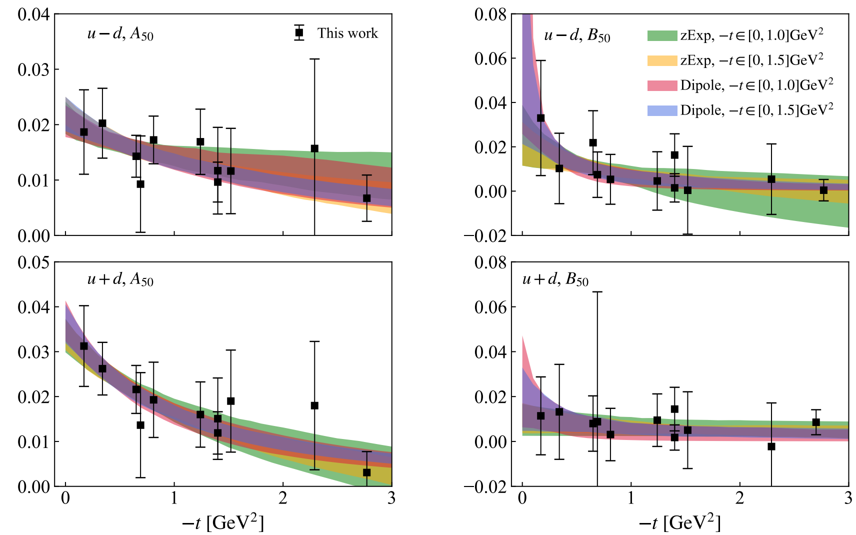
# Calculation of Moments of GPDs (L2.1)

Use Operator Product Expansion (OPE) of non-local operators at short distances to obtain *Mellin Moments of the GPDs - or Generalized Form Factors*

S.Bhattacharya *et al.*, Phys.Rev.D 108 (2023) 1 014507

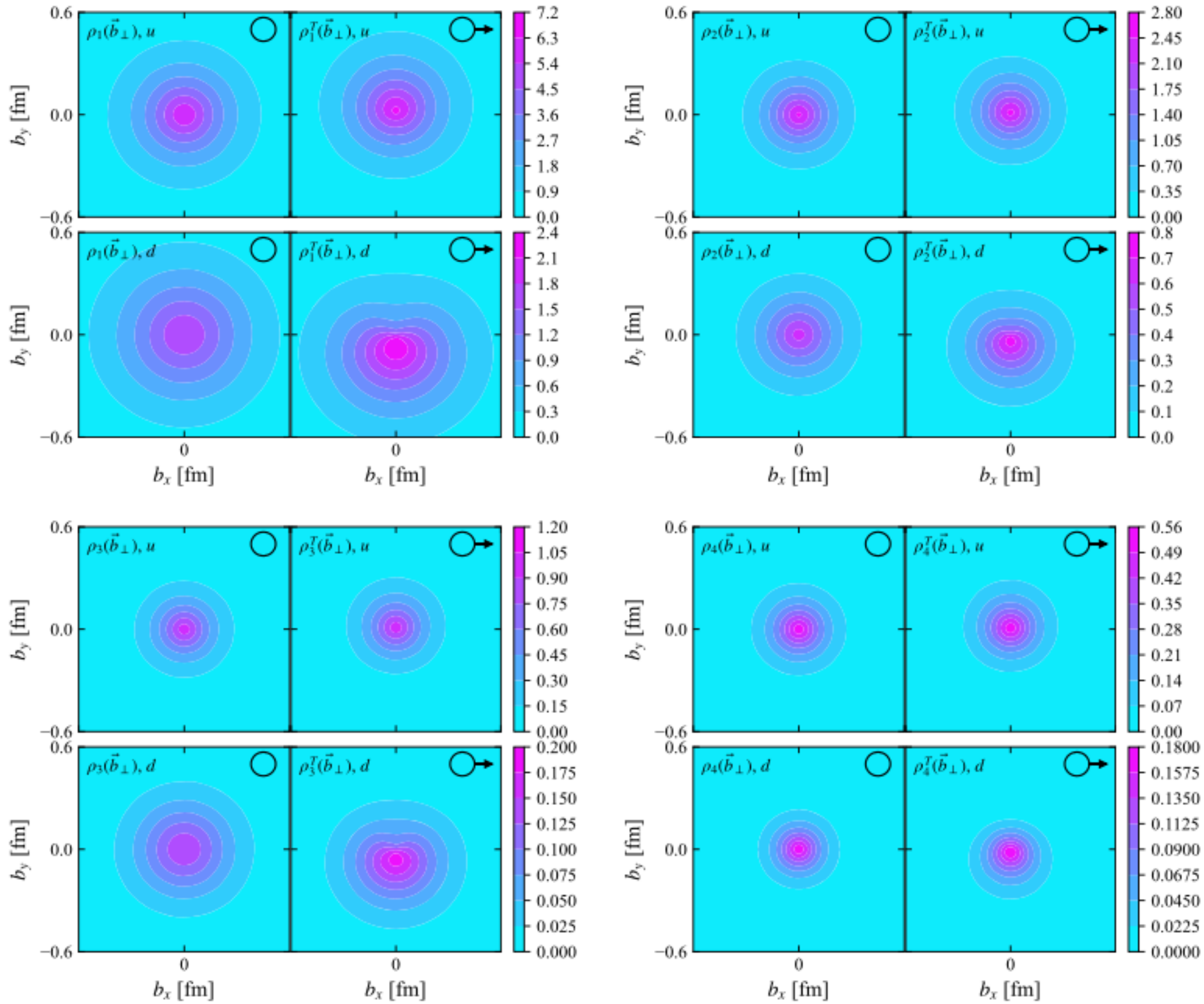



Extraction of higher moments precluded using local operators

# Moments of GPDs - II




Transform to impact-parameter space: *narrowing of distribution with increasing moment*

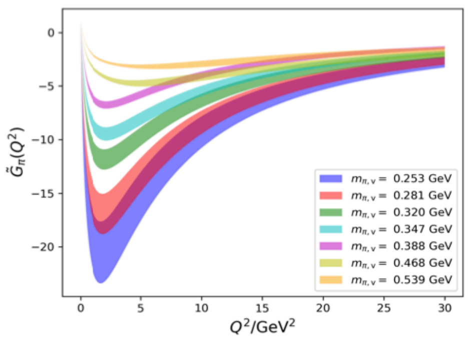
# Trace-Anomaly Form Factors (L3.1)

- Understanding the distribution of charge, spin and mass within a hadron is a fundamental goal of NP.
- Knowledge about internal mechanical properties encapsulated in the energy-momentum tensor

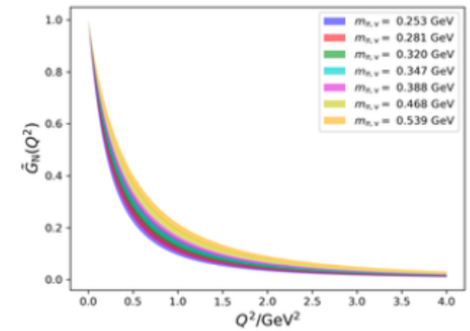
$$\hat{T}_g^{\mu\nu} = 2\,\mathrm{Tr}\left[-F^{\mu\alpha}F^{\nu}_{\alpha} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right];\ \hat{T}_q^{\mu\nu} = 2\sum_f i\bar{\psi}_f D^{\{\nu}\gamma^{\nu\}}\psi_f$$

Key role is played by the trace-anomaly and its form factors

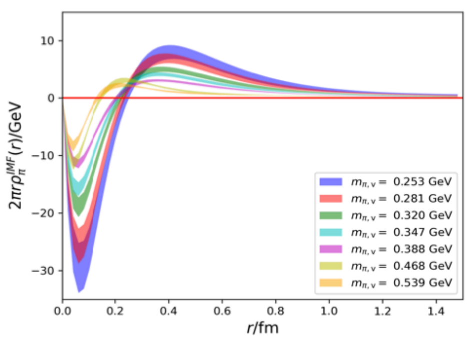
$\langle p'|T^{\mu}_{\mu}|p\rangle = m_\pi F(Q^2)$ *for case of pion, and the mass at* $Q^2 = 0.$




$\chi$QCD Collaboration, B.Wang *et al.*, arXiv:2401.05496, PRD (to appear)




Gluon trace anomaly in Pion and Nucleon

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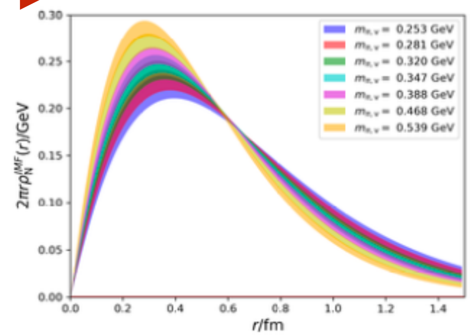


# Gravitational Form Factors (L4.3)

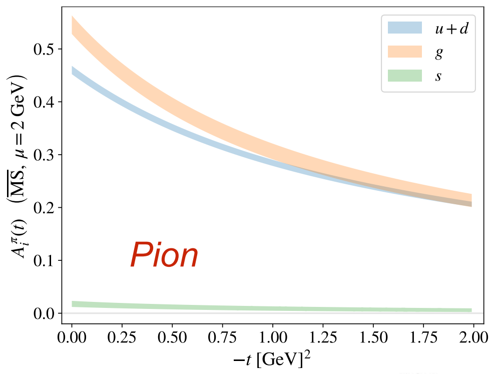
*Gravitational form factors are given by the matrix elements of the (symmetrised) energy-momentum tensor within a hadron.*

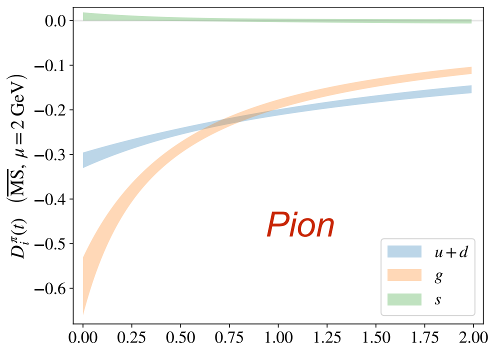
$$\hat{T}_g^{\mu\nu} = 2\,\mathrm{Tr}\left[-F^{\mu\alpha}F_\alpha^\nu + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right];\ \hat{T}_q^{\mu\nu} = 2\sum_f i\bar{\psi}_f D^{\{\nu}\gamma^{\nu\}}\psi_f$$

We can look at the Lorentz structure to learn more about the *mechanical* properties

For pion, they decompose as $\langle\pi(p') \mid \hat{T}^{\mu\nu} \mid \pi(p)\rangle = 2P^\mu P^\nu A^\pi(t) + \frac{1}{2}[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2]D^\pi(t)$

*Mechanical properties*
*Pressure distn*



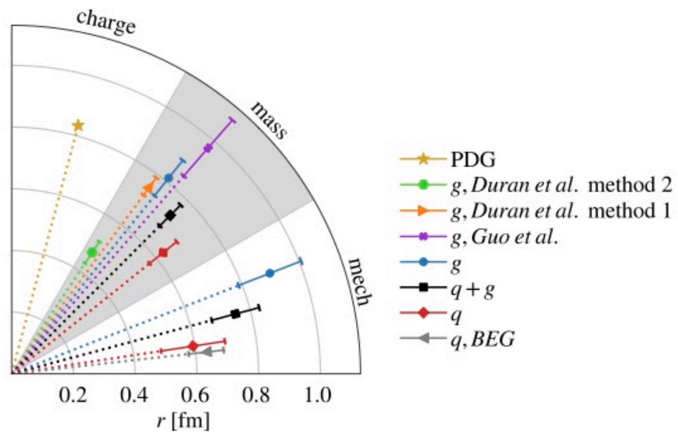




Clover-fermion computation at $m_\pi = 170\,\mathrm{MeV}$

*Complete flavor and gluon decomposition; renormalized to RI-mom*

Observation: smaller mass radius than charge radius

Proton “radii”




D.C.Hackett *et al*., Phys.Rev.D 108 (2023) 11, 114504

D.C.Hackett *et al*., arXiv:2310.08484

Joint Theory-Expt Review

V.Burkert et al., Rev. Mod. Phys. 95 (2023) 4, 041002

Requested DOE Highlight

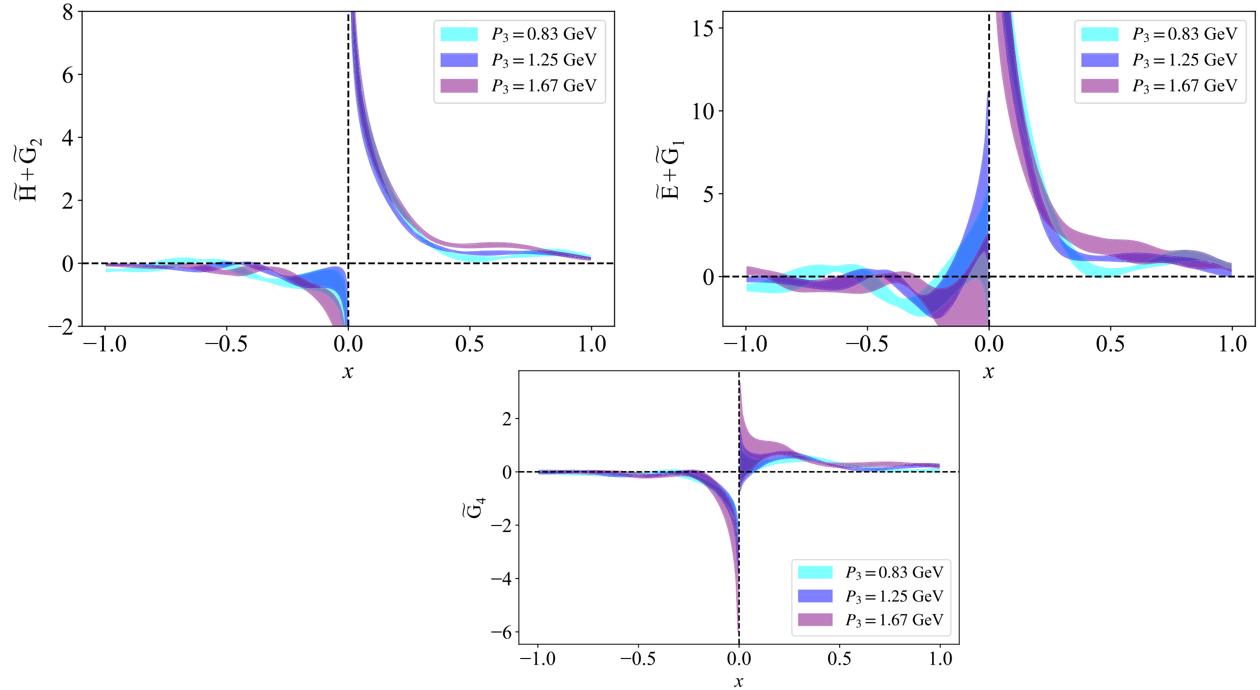
# Framework and Analysis of Twist-3 GPDs… (L4.2)

## First calculation of twist-3 axial GPDs within LaMET

S. Bhattacharya *et al.*, PHYSICAL REVIEW D 108, 054501 (2023)

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta) = \frac{1}{2P^+}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^+\gamma_5}{P^+}\tilde{H}(x,\xi,t) + P^\mu\frac{\Delta^+\gamma_5}{2mP^+}\tilde{E}(x,\xi,t) + \Delta_\perp^\mu\frac{\gamma_5}{2m}\Big(\tilde{E}(x,\xi,t)+\tilde{G}_1(x,\xi,t)\Big) + \gamma_\perp^\mu\gamma_5\Big(\tilde{H}(x,\xi,t)+\tilde{G}_2(x,\xi,t)\Big) + \Delta_\perp^\mu\frac{\gamma^+\gamma_5}{P^+}\tilde{G}_3(x,\xi,t) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^+}{P^+}\tilde{G}_4(x,\xi,t)\Big]u(p_i,\lambda).$$

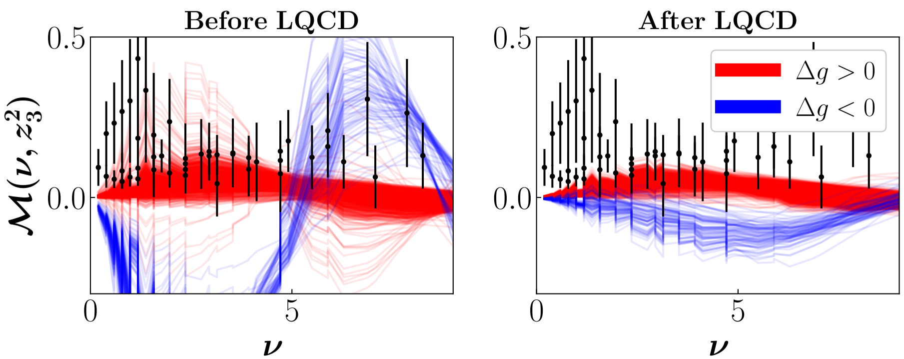
$\tilde{H}$ and $\tilde{E}$ occur at twist 2; $\tilde{G}_i$ at twist 3



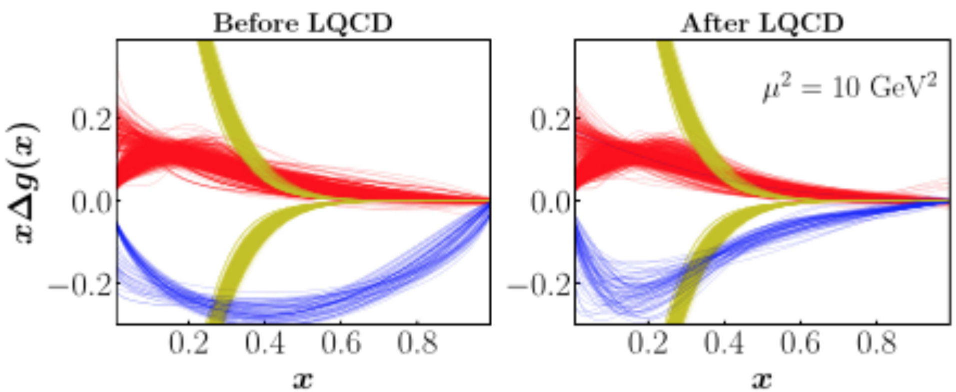
# Gluon Helicity Distribution

The culmination of QGT is a framework where LQCD + Expt can provide a more faithful description of hadron structure than either alone.

Does QCD admit negative solutions $\Delta g(x) < 0$

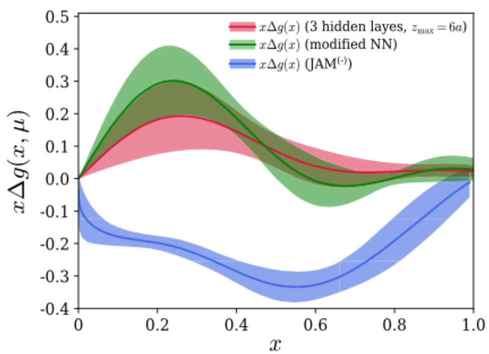



J.Karpie *et al.*, Phys.Rev.D 109 (2024) 3, 036031




Significant change in parametrization but insufficient to exclude negative solute in global analysis

Impact of LQCD on global analysis




Neural network analysis of **lattice calculation**

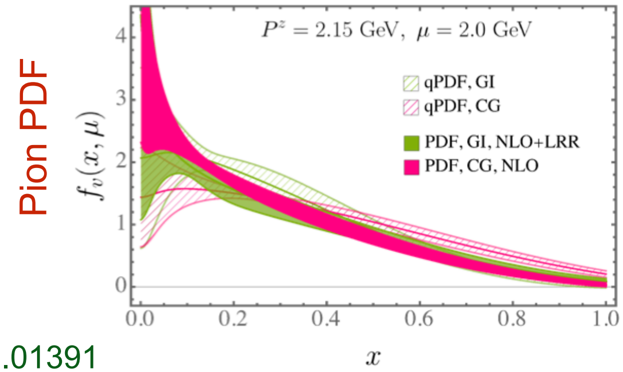
T.Khan,T.Liu and R.Sufian, Phys. Rev. D 108, 074502

RHIC polarized jet, large-x JLab + LQCD eliminate negative solutions: N.Hunt-Smith et al., arXiv:2403.08117

# Coulomb gauge Computations of Hadron Structure

Calculations in Coulomb Gauge (CG): higher off-axis momenta, absence of $1/a$ divergence in Wilson line,… compared to Gauge Invariant (GI) X.Gao, W-Y.Liu and Y.Zhao, arXiv:2306.14960 (PRD in press)

Applied first to pion quark PDF

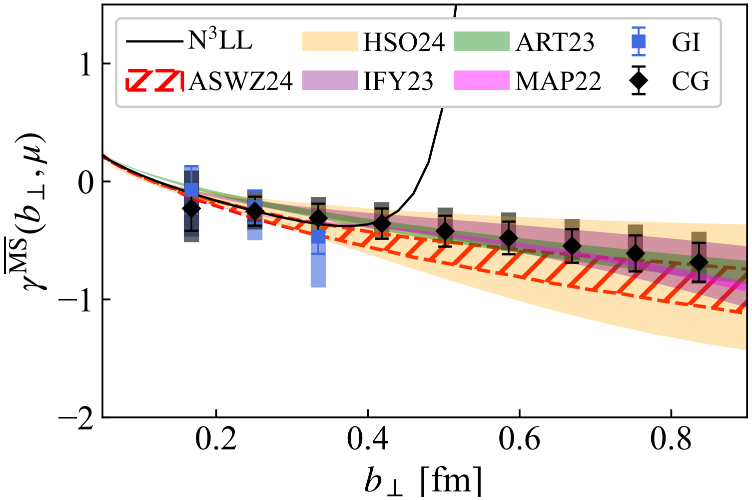



Application to quasi-TMDs: Y.Zhao, arXiv:2311.01391

Collins-Soper kernel from the Coulomb gauge (CG) approach

DWF Lattices at physical quark mass and fine lattice spacing

D.Bollweg, X.Gao, S.Mukherjee, Y.Zhao, Phys.Lett. B852 (2024) 138617




- *Significantly improved precision compared to the gauge-invariant (GI) approach*
- *Increased reach in transverse separations*
- *Good agreement with phenomenological determinations and recent GI calculation ASWZ24*

Anticipate applying widely to 3D structure: TMDs, GPDs, GTMDs

# Summary

= complete

= significant progress

**Year 1: L1.1 Pion DA and connection to DVMP - ANL (Yong Zhao)**

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15 Papers - *2 with experimental collaborators*

LQCD calculations that impact our understanding of tomography

Image courtesy https://icons8.com/