MSTT factorization: Bridging Large-x and Small-x

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Based on: S. Mukherjee (BNL), V. V. Skokov, A. Tarasov, S. T., "Unified description of DGLAP, CSS, & BFKL..."
Phys. Rev. D 109 (3) (2024) 034035

Main Advantage

One of SURGE Goals: Initial condition from Lattice QCD for small-x evolution

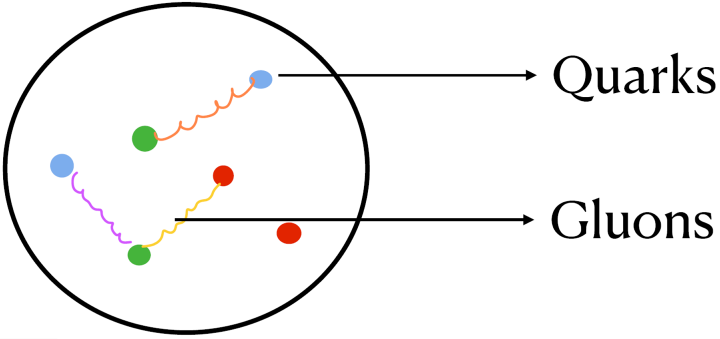
Problem

- **Objects of interest** cannot be directly computed on the lattice
- Compute **objects of interest** approximately
- Approximations **bad** at small-x

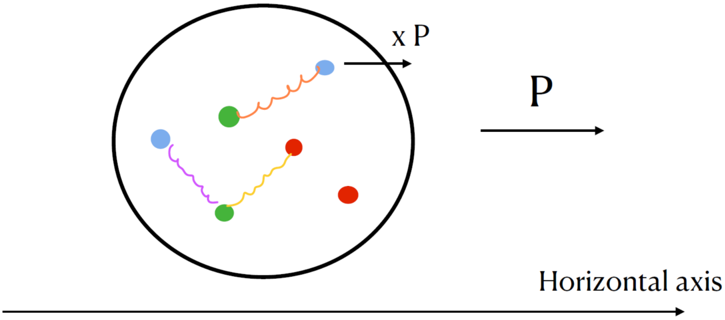
Solution

- **MSTT scheme** valid for all x
- Calculate **lattice results** using MSTT **at moderate** x
- **Evolve** to small-x

Structure of proton



One-dimensional structure of proton

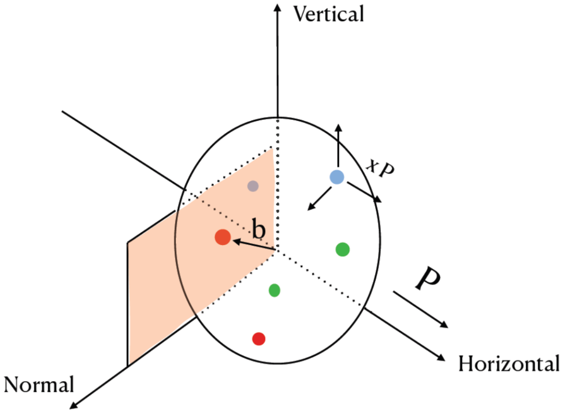


x is between 0 and 1

Parton Distribution Function (PDF)

$\mathrm{PDF}(x)$

Three dimensional structure of proton



Transverse Momentum Dependent (TMD) PDF

$$\text{TMDPDF}(x, b)$$

What is factorization?

$$\boxed{\text{Experimental Observables} \;=\; \text{Hard factor} \otimes \text{TMDPDF}(x, b, \Lambda)}$$

x : Longitudinal momentum fraction

b : Transverse structure

Λ : Scheme dependent parameters

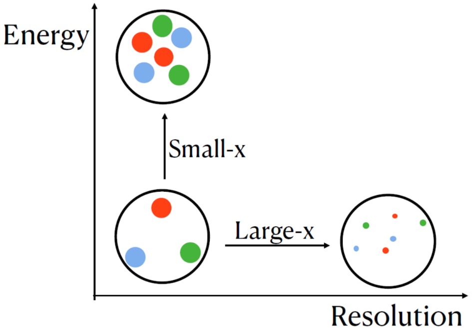
Evolution: Changing Λ

Previous works

Schemes

- **Large-x factorization**
  Get **Large-x evolution**

- **Small-x factorization**
  Get **Small-x evolution**



How do you connect the two regimes/evolutions?

New scheme with validity for all x is needed: MSTT

Large-x (N.P.B, 250, 1985)
Small-x (P.L.B, 60, 1975)

MSTT factorization

MSTT factorization is based on the background field method

gluon TMDPDF(x, b) operator

↓ Quantum corrections

TMDPDF(x, b, Λ) valid for all x

Small-x limit ↙ ↘ Large-x limit

Small-x factorization/BFKL **Large-x factorization/CSS**

Large-x (N.P.B, 250, 1985)
Small-x (P.L.B, 60, 1975)
MSTT (P.R.D 109, no.3, 034035 (2024))

Main Advantage

One of SURGE Goals: Initial condition from Lattice QCD for small-x evolution

Problem

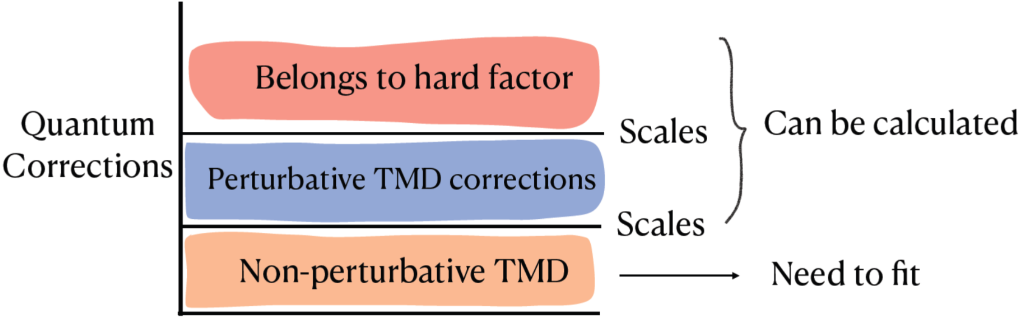
- TMDPDFs cannot be directly computed on the lattice
- Instead compute Quasi-TMDPDFs and get TMDPDFs (approximately)
- Approximations bad at small-x

Solution

- MSTT valid for all x
- Lattice results from moderate x and evolve to small-x

Methodology: Background field method

Main point: Separate Quantum corrections using Background field method



L. F. Abbott. The Background Field Method Beyond One Loop. Nucl. Phys. B, 185:189–203, 1981.

Full result

The full result contains parts of Large-x and parts of Small-x

$$f_{ij}(x,b_\perp,\mu_{\rm UV}^2,\zeta) = f_{ij}(x,b_\perp,\mu_{\rm IR}^2,\rho) - 4\alpha_s N_c \int \bar{d}^2 p_\perp e^{ip_\perp b_\perp} \int_0^1 \frac{dz}{z(1-z)} \int \bar{d}^2 k_\perp \Big[\mathcal{R}^a_{ij;lm}(z,p_\perp,k_\perp)$$

$$+\mathcal{R}^b_{ij;lm}(z,p_\perp,k_\perp)\Big] \int d^2 z_\perp e^{-i(p_\perp - k_\perp)z_\perp} f_{lm}(\frac{x}{z},z_\perp,\mu_{\rm IR}^2,\rho) + \boxed{\frac{\alpha_s N_c}{2\pi}\Big(-\frac{1}{2}(L_b^{\mu_{\rm UV}})^2 + L_b^{\mu_{\rm UV}} \ln\frac{\mu_{\rm UV}^2}{\zeta^2} - \frac{\pi^2}{12}\Big)}$$

$$\times f_{ij}(x,b_\perp,\mu_{\rm IR}^2,\rho) - \boxed{\frac{\alpha_s N_c}{\pi} L_b^{\mu_{\rm IR}} \int_0^1 dz \Big[\frac{1}{(1-z)_+} + \frac{1}{z}\Big]} f_{ij}(\frac{x}{z},b_\perp,\mu_{\rm IR}^2,\rho) - \frac{\alpha_s N_c}{2\pi} \int d^2 z_\perp \int \bar{d}^2 p_\perp e^{ip_\perp (b-z)_\perp}$$

$$\times\Big(\frac{1}{2}\ln^2\frac{\mu_{\rm IR}^2}{p_\perp^2} + \boxed{\ln\frac{\mu_{\rm IR}^2}{p_\perp^2}\ln\frac{\rho}{\zeta}} - \frac{\pi^2}{12}\Big)\frac{g_{il}p_j p_m + p_i p_l g_{mj}}{p_\perp^2} f_{lm}(x,z_\perp,\mu_{\rm IR}^2,\rho)$$

$$+\frac{\alpha_s N_c}{2\pi} \int d^2 z_\perp \int \bar{d}^2 p_\perp e^{ip_\perp (b-z)_\perp}\Big(\frac{\beta_0}{2N_c}\ln\frac{\mu_{\rm UV}^2}{p_\perp^2} + \frac{67}{18} - \frac{5N_f}{9N_c}\Big) f_{ij}(x,z_\perp,\mu_{\rm IR}^2,\rho) + O(\alpha_s^2)\,.$$

$$\boxed{\zeta = \text{constant} \times x}$$

Summary and Outlook

- MSTT factorization bridges large and small-x regimes; serves as a tool to connect Lattice QCD to small-x observables at Electron Ion Collider

- Our approach is based on the background field method which naturally allows us to separate and calculate the quantum corrections

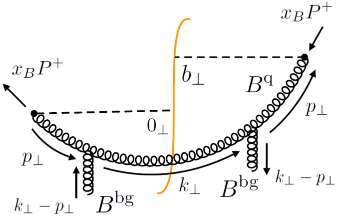
Outlook

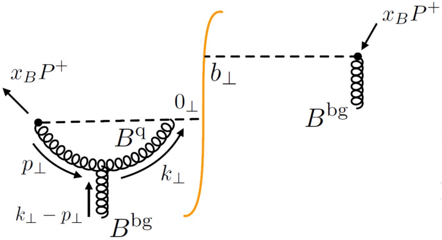
- Extension to Lattice operators
- Extension to Quark TMDPDFs

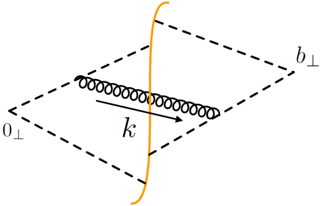
Swagato Mukherjee, "A unified description of DGLAP, CSS, & BFKL", DIS 2024, April 10th
Shaswat Tiwari, "MSTT factorization", QCD evolution workshop 2024, May 27th to May 31st

Back-up slides

Representative diagrams







Methodology: Definitions

$$B_{ij}(x, b_\perp) = \int_{-\infty}^{\infty} dz^-\, e^{-ixP^+z^-} \langle P, S|\, \bar{T}\{F^m_{-i}(z^-, b_\perp)\, W[z^-, \infty]^{ma}_b\} T\{W[\infty, 0^-]^{an}_0 F^n_{-j}(0^-, 0_\perp)\}|P, S\rangle$$

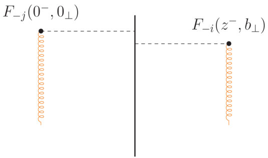


Figure 1: Beam function

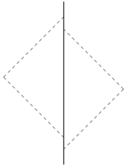


Figure 2: Soft function

$$\mathcal{S}(b_\perp) = \langle 0| Tr\Big[S^\dagger_{\bar{n}}(b_\perp)\, S_n(b_\perp)\, S^\dagger_n(0_\perp) S_{\bar{n}}(0_\perp)\Big]|0\rangle$$

With,

$$f = B\sqrt{S}$$

Divergences

We isolate and regulate the divergences using the following methods,

- Dimensional Regularization: $k_\perp \to \infty(\mu_{UV})$, $k_\perp \to 0\ (\mu_{IR})$
- η Regularization : $k^- \to \infty(\nu_{UV})$, $\boxed{\mathrm{k}^- \to 0(\rho_{IR})}$: **New**

$$\int_0^\infty \frac{dk^-}{k^-} \to \int_0^\infty \frac{dk^-}{k^-}|k^+|^{-\eta}$$

Different Limits

- Collinear limit: Take $p_\perp - k_\perp \to 0$

$$f_1(x, b_\perp, \mu_{\rm UV}^2, \zeta) = f_1(x, 0_\perp, \mu_{\rm IR}^2)$$

$$- \underbrace{\boxed{\frac{\alpha_s N_c}{\pi} L_b^{\mu_{\rm IR}} \int_0^1 \frac{dz}{z} P_{gg}(z) f_1(\frac{x}{z}, 0_\perp, \mu_{\rm IR}^2)}}_{DGLAP} + \underbrace{\boxed{\frac{\alpha_s N_c}{2\pi}\Big(-\frac{1}{2}(L_b^{\mu_{\rm UV}})^2 + L_b^{\mu_{\rm UV}} \ln\frac{\mu_{\rm UV}^2}{\zeta^2} - \frac{\pi^2}{12}\Big) f_1(x, 0_\perp, \mu_{\rm IR}^2)}}_{CSS} + \dots,$$

- Small-x limit: Take $x \to 0$,

$$f_1(x, p_\perp, \mu_{\rm UV}^2, \zeta) \simeq \mathcal{H}_1(p_\perp, \rho) + \boxed{\ln\frac{\rho}{\zeta} \int \bar{d}^2 k_\perp K_{\rm BFKL}(p_\perp, k_\perp) \mathcal{H}_1(p_\perp - k_\perp, \rho)}$$

$$+ \frac{\alpha_s N_c}{2\pi} \int d^2 b_\perp \boxed{\Big(-\frac{1}{2}(L_b^{\mu_{\rm UV}})^2 + L_b^{\mu_{\rm UV}} \ln\frac{\mu_{\rm UV}^2}{\zeta^2} - \frac{\pi^2}{12}\Big)} \int \bar{d}^2 k_\perp e^{ik_\perp b_\perp} \mathcal{H}_1(p_\perp - k_\perp, \rho)$$

$$+ \frac{\alpha_s N_c}{2\pi}\Big(\frac{\beta_0}{2N_c} \ln\frac{\mu_{\rm UV}^2}{p_\perp^2} + \frac{67}{18} - \frac{5N_f}{9N_c}\Big) \mathcal{H}_1(p_\perp, \rho).$$